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FERMILAB COORDINATE SYSTEMS

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ABSTRACT

The objectives of this document are to relate Fermilab to a global coordinate system, to define a mapping projection and establish all the coordinate systems that will be used at Fermilab and for the Fermilab Main Injector (FMI) project, and to describe all the parameters that are necessary for the several coordinate systems. The main objective is to define a new **assimilated** DUSAF coordinate system for the Fermilab site. This implies that the new system will have the same origin and coordinate axes definitions as the DUSAF coordinate system. The Survey Alignment and Geodesy (SAG) group, has defined a **Fermilab Site Coordinate System** (**FSCS**) for the site and a **Local Tunnel Coordinate System** (**LTCS**) for the FMI project. A **Double Stereographic Projection** has been adopted to define both coordinate systems.

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FERMILAB COORDINATE SYSTEMS

1. Introduction

Several years ago a DUSAF coordinate system was established at Fermilab and the original documentation could not be traced. It is a Cartesian coordinate system with no known map projection associated with it. Therefore, it is not possible to directly convert these DUSAF plane coordinates to the global geodetic (or geographic) coordinate system using the appropriate Earth parameters. The need for a map projection and a tie to a global coordinate system also stems from the future global experiments being planned at Fermilab. These global experiments use the Global Positioning Systems (GPS) technology in which data is collected in the geodetic coordinate system.

The objectives of this document are to relate Fermilab to a global coordinate system, to define a mapping projection and establish all the coordinate systems that will be used for the Fermilab site and for the Fermilab Main Injector (FMI) project, and to describe all the parameters that are necessary for the several coordinate systems.

Since all lattice coordinates for all Fermilab and FMI projects are provided in the DUSAF coordinate system, the main objective is then to define a new **assimilated** DUSAF coordinate system for the Fermilab site. This implies that the new system will have the same origin and coordinate axes definitions as the DUSAF coordinate system.

1.1 DUSAF Coordinate System

The DUSAF coordinate system is a right-handed Cartesian coordinate system defined as follows:

Origin - A0
Y-axis - NORTH axis. Positive along the extraction line towards the Neutrino Area

X-axis - EAST axis. Positive to the right and perpendicular to the y-axis

Z-axis - ELEVATION axis. Positive up at A0 and perpendicular to both X- and Y-axes.

The parameters that define the DUSAF coordinate system origin are:

X	=	100000.000 ft	(30480.06096 m)	; False Easting at A0
Y	=	100000.000 ft	(30480.06096 m)	; False Northing at A0
Z	=	720.000 ft	(219.45644 m)	; Elevation

The DUSAF elevation Z is referenced to the DUSAF (Vertical) Datum which is an arbitrary datum. At A0, Z = 720.000 ft implies that the elevation of A0 is 720 ft above the DUSAF Datum.

1.2 Map Projection

The Survey Alignment and Geodesy group has adopted a **Double Stereographic Projection** (see Appendix D) to define a **Fermilab Site Coordinate System (FSCS)** for the Fermilab site. The **FSCS** is referenced to the North American Datum of 1983 (NAD83) and its reference ellipsoid of 1980 (GRS80). The Double Stereographic Projection was chosen to map the GRS80 geodetic coordinates onto the conformal mapping plane which defines the Fermilab Site Coordinate System. The Double Stereographic Projection was also adopted to define a **Local Tunnel Coordinate System (LTCS)** for the FMI project.

Before these new coordinate systems and other projection systems are defined in greater details, the **Geodetic Coordinate System (GCS)** will be described. The geodetic coordinate system is the fundamental coordinate system with respect to which all other coordinate systems are defined.

2. Geodetic Coordinate System (GCS)

GCS is a curvilinear coordinate system, based upon a geocentric, bi-axial reference ellipsoid (GRS80), called the North American Datum of 1983 (NAD83), with axes coincident with those of the Geodetic Cartesian Coordinate System, axis of rotation along the Z-axis (see section 3 below). It is the fundamental coordinate system with respect to which all other coordinate systems are defined. The geometric parameters of the ellipsoid are:

a = 6378137.000 m (exactly, by definition)
$1/f$ = 298.257 222 101 (to 12 significant digits)

where a is the semi-major axis of the ellipsoid and f is the flattening of the ellipsoid.

The coordinates of a given point, P_i, within this system are defined as:

ϕ_i = Geodetic Latitude
λ_i = Geodetic Longitude
h_i = Ellipsoidal Height (height above the ellipsoid, normal to the ellipsoidal surface)

Since the ellipsoidal height is directly related to the orthometric height (elevation), this relationship will be described in the following section.

2.1 Orthometric Height

The orthometric height, H, of a point, P, is defined as the geometric distance between the geoid and the point, measured along the plumb line through the point. The orthometric height is based on the North American Vertical Datum of 1988 (NAVD88). The orthometric height above the NAVD88 datum is determined by

$$H_{NAVD88} = h - N$$

where

h = Ellipsoidal Height, as defined in the Geodetic Coordinate System

N = Geoid Height, the separation between the Reference Ellipsoid and the Geoid at point, P.

The geoid height values are interpolated from a grid of regularly-spaced estimates using the GEOID93 Model based on GRS80 (or WGS84, which is equivalent to NAD83, as defined below). The actual computation is performed as an interpolation from a regularly-spaced grid of points. The interpolation is accomplished by a locally fit biquadratic function. The polynomial surface is fit to nine data points defining the area surrounding the point where the interpolation is to take place.

The NAVD88 orthometric heights are derived from heights above the DUSAF datum using the expression:

$$H_{NAVD88} = H_{DUSAF} + dH_{NAVD88}$$

where $dH_{NAVD88} = -0.17308$ m is the correction to tie the DUSAF datum to the NAVD88 datum.

The relationship between the ellipsoidal height in the **GCS** system and the orthometric height above the DUSAF datum is given by:

$$h = H_{DUSAF} + dH_{NAVD88} + N$$

3. Geodetic Cartesian Coordinate System (GCCS)

GCCS is a geocentric, right handed Cartesian coordinate system, with axes defined to be coincident with those of the Reference Ellipsoid GRS80, used for NAD83 datum. It is used as an intermediate system by

which other systems are converted to and from the geodetic coordinate system **GCS**. The orientation is defined as follows:

Z-axis - Parallel to the direction of the BTS-84 Terrestrial Pole
BTS-84 = Bureau International de l'Heure (BIH) Terrestrial System of 1984
X-axis - Parallel to the intersection of the BTS-84 Reference Meridian and the plane of the equator of the BTS-84 pole
Y-axis - Perpendicular to both X- and Z-axes, to complete the right handed coordinate system.

In these respects, NAD83 is similar to the other modern global reference system, such as the World Geodetic System of 1984 (WGS84). The Global Positioning Systems (GPS) data are given in the WGS84 coordinate system. In principle, the three dimensional coordinates of a single physical point should be the same in both systems; in practice, small differences are sometimes found. For these coordinate system definitions and transformations, it can be assumed for all practical purposes that the NAD83 and WGS84 are entirely coincident. There are standard formulas used for conversion between the **GCCS** coordinates (XYZ) and the geodetic coordinates (ϕ, λ,h) of the **GCS**. Transformation between the **GCCS** coordinates (XYZ) and other Cartesian coordinate systems are also possible.

4. Fermilab Site Coordinate System (FSCS)

The Fermilab Site Coordinate System (FSCS) is an **assimilated** DUSAF coordinate system. Its origin and rotation axes are located at A0, preserving as nearly as possible the DUSAF coordinate system.

4.1 FSCS:XYZ

FSCS:XYZ coordinate system corresponds to the lattice version of the **FSCS**. This coordinate system was developed for the beamlines extracting from the Main Ring and the Tevatron at A0. It is a right-handed Cartesian coordinate system defined as follows:

- Origin - A0
- Y-axis - NORTH axis rotated by Geodetic Azimuth α from the Geodetic North. Positive along the extraction line towards the Neutrino Area
- X-axis - EAST axis. Positive to the right and perpendicular to the Y-axis
- Z-axis - Positive up and perpendicular to both X- and Y-axis. It is aligned with the direction of the normal to the ellipsoid at A0.

The parameters that define the **FSCS:XYZ** coordinate system origin at A0 are:

X	=	100000.000 ft	(30480.06096 m)
Y	=	100000.000 ft	(30480.06096 m)
Z	=	720.000 ft	(219.45644 m)

To relate the **FSCS:XYZ** system to the **GCS** system, the ellipsoidal height of A0 is given as:

$$h_0 = 186.49880 \text{ m}$$

The Geodetic Azimuth α defining the alignment of the Y-axis at A0 is given as:

$$\alpha = 38° \, 16' \, 48.01429''$$

4.2 FSCS:XYH

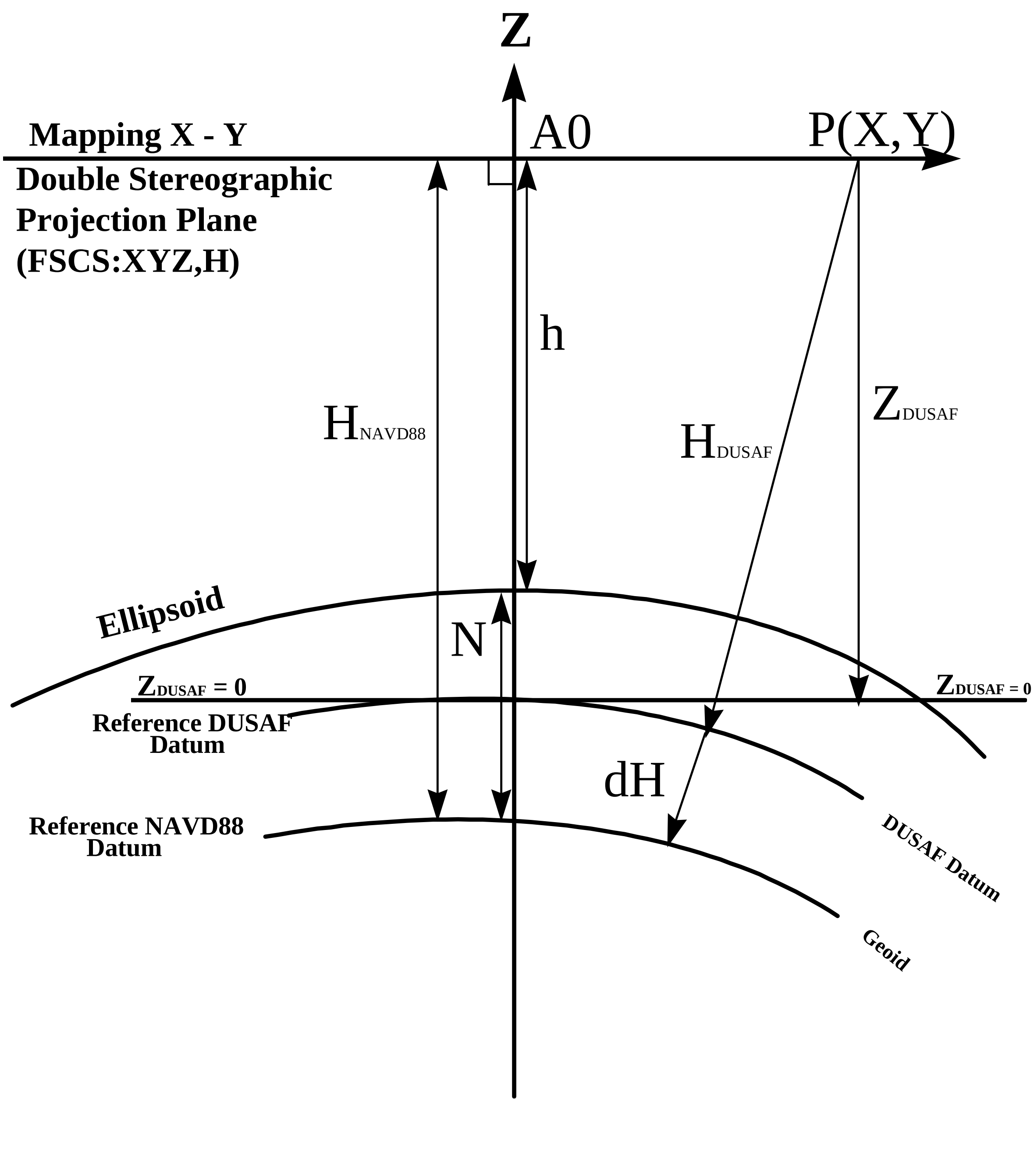
FSCS:XYH is a right-handed Cartesian coordinate system based on a Double Stereographic Projection and heights above the DUSAF datum. At the origin A0, the (XYH) coordinates in this system are identical to the (XYZ) coordinates in the **FSCS:XYZ** system. Figure 1 shows the relationship between the **FSCS:XYZ** and **FSCS:XYH** systems. **FSCS:XYH** is a two dimensional mapping plane whose coordinates (X,Y) or (E, N; Easting, Northing) are generated by a Double Stereographic Projection of geodetic coordinates (ϕ,λ), with the origin defined as the point on the ellipsoid corresponding to A0. The Double Stereographic Projection is performed in two steps: namely, the projection from the reference ellipsoid to a conformal sphere and from the sphere to a plane.

The basic parameters for performing the Double Stereographic Projection are as follows:

ϕ_0 = Geodetic Latitude of the origin
λ_0 = Geodetic Longitude of the origin
h_0 = Ellipsoidal Height of the origin
X_0 = False Easting at the origin
Y_0 = False Northing at the origin
F_0 = Scale factor at the origin
α = Geodetic Azimuth of the Y-axis (North) at the origin

The geodetic coordinates (ϕ,λ) are projected into E and N using the double Stereographic projection, with the origin defined as the point on the ellipsoid corresponding to A0. The E and N coordinates are rotated about the Z-axis by the angle α (geodetic azimuth) to obtain the X and Y coordinates. The X and Y coordinates are then re-scaled by the scale factor F_0 to give true scale at the origin at 720 ft above the DUSAF datum. False coordinates are applied at the origin.

Figure 1. Fermilab Site Coordinate Systems (FSCS)



The defined parameters of the Double Stereographic Projection for the **FSCS:XYH** are given below:

Geodetic coordinates at the origin A0

$$\phi_0 = \text{N } 41° \, 50' \, 14.312704''$$
$$\lambda_0 = \text{W } 88° \, 15' \, 41.143123''$$
$$h_0 = 186.49880 \text{ m}$$

The relationship between the ellipsoidal height h_0 in the **GCS** system and the orthometric height above the DUSAF datum is given by:

$$h_0 = H_{720DUSAF} + dH_{NAVD88} + N_0$$

where $H_{720DUSAF}$ corresponds to 720 feet above the DUSAF datum and dH_{NAVD88} = -0.17308 m is the correction to tie the DUSAF datum to the NAVD88 datum. N_0 is the geoid height (geoid93 model) at A0 and is equal to -32.78456 m. At any other point the ellipsoidal height is given as:

$$h = H_{DUSAF} + dH_{NAVD88} + N$$

The orthometric height at A0 is given by:

$$H_{720NAVD88} = h_0 - N_0$$

The orthometric height at A0 in the **FSCS** = 219.45644 - 0.17308 = 219.28336 m. This relationship must be considered when transforming between the **FSCS** and other coordinate systems.

The orthometric heights are converted to heights above the DUSAF datum using the expression:

$$H_{DUSAF} = H_{NAVD88} - dH_{NAVD88.}$$

Geodetic Azimuth of the Y-axis of **FSCS** at the origin A0 is given by:

$$\alpha = 38° \, 16' \, 48.01429''$$

False coordinates applied at the origin are given by:

X_0 = 100000.000 ft (30480.06096 m) ; False Easting at A0
Y_0 = 100000.000 ft (30480.06096 m) ; False Northing at A0

At A0, the scale factor corresponding to the height of 720 ft above the DUSAF datum ($H_{720DUSAF}$) is given by:

F_0 = 1.000029251309483

5. Local Tunnel Coordinate System (LTCS)

The Local Tunnel Coordinate System (LTCS) was established to meet the stringent accuracy requirements of the FMI project. Its origin and rotation axes are located at a point, CFMI, which lies at the centroid of the FMI Plane and whose (X,Y) coordinates are in the **FSCS**. The coordinates in the LTCS are defined in the FMI Plane. The FMI Plane is that plane defined by the nominal designed orthometric heights of the cell boundaries 308, 522, and 620. The coordinates of the cell boundaries 308, 522, and 620 in the FMI Plane (and other coordinate systems) are given in Table A1 in Appendix A. To convert the LTCS coordinates to global geodetic coordinates the FMI Plane must be tilted by a small angle to a projection plane

5.1 LTCS:XYZ

The **LTCS:XYZ** coordinate system corresponds to the lattice version of the **LTCS**. The X-Y plane of the **LTCS** is coincident with the FMI plane. It is a right-handed Cartesian coordinate system defined as follows:

Origin - CFMI (Centroid of the FMI Plane)
Y-axis - NORTH axis rotated by Geodetic Azimuth α from the Geodetic North.
X-axis - EAST axis. Positive to the right and perpendicular to the Y-axis.

Z-axis - Positive up at CFMI and perpendicular to both X- and Y-axes.

The parameters that define the **LTCS:XYZ** coordinate system origin at CFMI are:

X =	100,661.49800 ft	(30681.68595 m)
Y =	95,856.98802 ft	(29217.26838 m)
Z =	715.664 ft	(218.13491 m)

To relate the **LTCS:XYZ** system to the **GCS** system, the ellipsoidal height of CFMI is given as:

$$h_0 = 185.19035 \text{ m}$$

The Geodetic Azimuth α defining the alignment of the Y-axis at CFMI is given as:

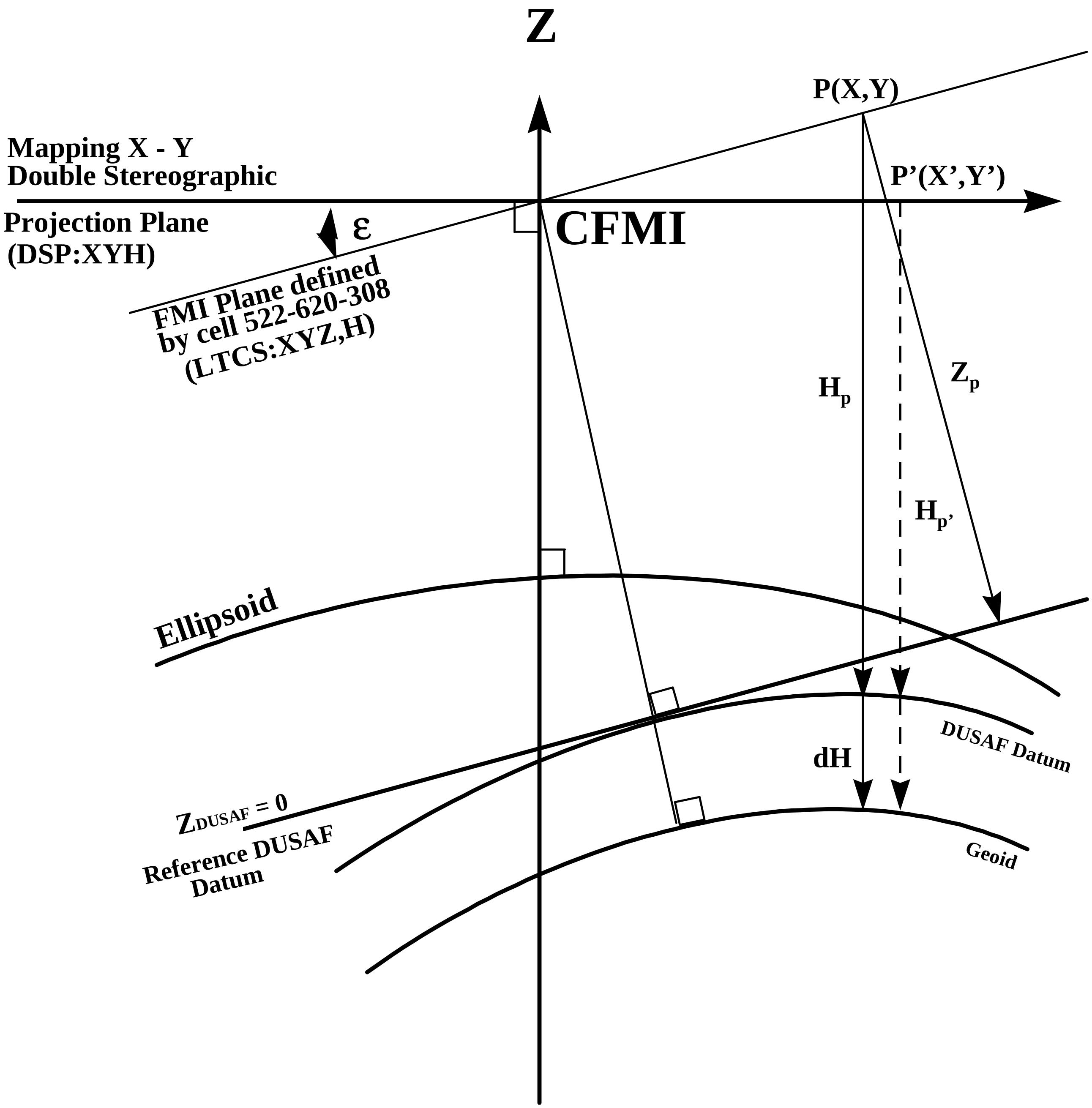
$$\alpha = 38° \, 16' \, 29.97831''$$

5.2 DSP:XYH

DSP:XYH is a right-handed Cartesian coordinate system based on Double Stereographic Projection and heights above the DUSAF datum. The (XYH) coordinates in this system are identical to the (XYZ) coordinates in the **LTCS:XYZ** system at the origin CFMI. The relationship between the **LTCS:XYZ** and **DSP:XYH** systems is shown in Figure 2.

DSP:XYH is a two dimensional mapping projection plane whose coordinates (X,Y) or (E, N; Easting, Northing) are generated by a Double Stereographic Projection of geodetic coordinates (ϕ,λ), with the origin defined as the point on the ellipsoid corresponding to CFMI. This projection plane is tilted from the **LTCS:XYH** FMI plane by an angle ε, see Figure 2. The basic parameters of the projection are the same as those of **FSCS:XYH**. The parameters for FMI have been selected to minimize

Figure 2. Local Tunnel Coordinate Systems (LTCS)



the effect of the point scale factor for all points around the FMI tunnel. The parameters of the projection are given below.

The geodetic coordinates (ϕ,λ) are projected into E and N using the double Stereographic projection, with the origin defined as the point on the ellipsoid corresponding to CFMI. The E and N coordinates are rotated about the Z-axis by the angle α (geodetic azimuth) to obtain the X and Y coordinates. The X and Y coordinates are then re-scaled by the scale factor F_0 to give true scale at the origin at 715.664 ft above DUSAF datum. False coordinates are applied at the origin. The defined parameters of the Double Stereographic Projection for the **DSP:XYH** are given below:

Geodetic coordinates at the origin at CFMI (Centroid of the FMI Plane):

ϕ_0 = N 41° 49′ 38.134927″
λ_0 = W 88° 16′ 08.184535″
h_0 = 185.19035 m

The ellipsoidal height in the **GCS** system, h, is converted to an orthometric height above the DUSAF datum using the same definitions and parameters as those given for the **FSCS:XYH** coordinate height.

Geodetic Azimuth of the Y-axis of **LTCS** at the origin CFMI is given by:

α = 38° 16′ 29.97831″

False coordinates applied at the origin are given by:

X = 100,661.49800 ft (30681.68595 m) ; False Easting at CFMI
Y = 95,856.98802 ft (29217.26838 m) ; False Northing at CFMI

At CFMI , the scale factor corresponding to the height of CFMI above the DUSAF datum ($H_{CFMIDUSAF}$) is given by:

F_0 = 1.000029046120306

5.3 LTCS:XYH

The actual working plane of reference for the FMI tunnel is the FMI Plane. Therefore the coordinates in the **DSP:XYH** projection plane must be related to the FMI Plane. The **LTCS:XYH** coordinates in the Lattice Program (see Appendix E) refers to the **LTCS:XYH** coordinates in the FMI Plane. The relationship between the **LTCS:XYZ, DSP:XYH** and **FSCS:XYH** systems is shown in Figure 2.

To rotate the coordinates on the **DSP:XYH** projection plane to the FMI Plane a seven-parameter transformation is performed using the following expression:

$$\underline{\mathbf{X}}_{\mathrm{LTCS:XYH}} = \underline{\mathbf{X}}_{\mathrm{Translation}} + S * \mathbf{R}(\varepsilon_X,\varepsilon_Y,\varepsilon_H) * \underline{\mathbf{X}}_{\mathrm{DSP:XYH}}$$

where $\underline{\mathbf{X}}_{\mathrm{LTCS:XYH}}$ is the vector containing the (XYH) coordinates in the FMI Plane; $\underline{\mathbf{X}}_{\mathrm{DSP:XYH}}$ is the vector containing the (XYH) coordinates in the projection plane; $\underline{\mathbf{X}}_{\mathrm{Translation}}$ is the vector containing the translation parameters in XYH; $\mathbf{R}(\varepsilon_X,\varepsilon_Y,\varepsilon_H)$ is the rotation matrix; and S is the scale.

The transformation parameters from the **DSP:XYH** projection plane to the **LTCS:XYH** FMI Plane are defined as follows:

Scale	=	1.00000		Fixed
X Rotation, ε_X	=	2.07594	arcsec	
Y Rotation, ε_Y	=	-0.74326	arcsec	
H Rotation, ε_H	=	0.00000	arcsec	Fixed
X Translation	=	-0.00079	m	
Y Translation	=	-0.00220	m	
H Translation	=	0.38636	m	

6. Local Geodetic System (LGS)

A left handed Cartesian coordinate system, defined to be topocentric about a point, P, i.e., a coordinate system with the origin at a point, P, of known geodetic coordinates. The left-handed coordinate system is defined as
follows:

h-axis - Outward ellipsoid normal in the **GCS** system passing through
point P.
n-axis - Perpendicular to the h-axis, and directed towards Geodetic North (i.e., lies in the plane defined by the Geodetic Cartesian Z-axis and point P).
e-axis - Perpendicular to both the n- and h-axes, forming a left handed coordinate system with the positive direction to the East.

It should be noted that there exists an infinite number of local geodetic coordinate systems, dependent upon the choice of point P.

7. Oblique Mercator Projection System (OMPS)

The Global Positioning Systems (GPS) data collected at the Fermilab are in the geodetic coordinate system. Present software available transform these data into a conformal mapping plane using the Oblique Mercator Projection and the same basic parameters as defined for the Double Stereographic Projection.

The Oblique Mercator Projection Coordinate System (**OMPS**) is a two dimensional mapping plane whose coordinates (X,Y) or (E, N), are generated by a Oblique Mercator Projection of Geodetic Latitude and Longitude (ϕ,λ) in the **GCS** system.

The defined parameters of the Oblique Mercator Projection for the Fermilab site are given below:

Geodetic Origin at A0

ϕ_0 = N 41° 50′ 14.312704″

λ_0 = W 88° 15′ 41.143123″

h_0 = 186.49880 m

The ellipsoidal height in the **GCS** system, h, is converted to an orthometric height above the DUSAF datum using the same definitions and parameters as those given for the **FSCS:XYH** coordinate height.

Geodetic Azimuth of the Central Meridian (center line) of the projection at the origin A0 is given by:

α = 38° 16′ 48.01429″

False coordinates applied at the origin are given by:

X_0 = 100000.000 ft (30480.06096 m) ; False Easting at A0

Y_0 = 100000.000 ft (30480.06096 m) ; False Northing at A0

At A0, the scale factor corresponding to the height of 720 ft above the DUSAF datum ($H_{720DUSAF}$) is given by:

F_0 = 1.000029251309483

8. Illinois State Plane System (ISPS)

It might be necessary sometime to relate the FSCS to an outside monument coordinates defined by the Illinois State Plane Coordinate System (**ISPS**). The Illinois State Plane Coordinate System is a two dimensional mapping plane whose coordinates (x, y) or (E, N), are generated by a 1983 State Plane Transverse Mercator Projection of Geodetic Latitude and Longitude (ϕ,λ) in the **GCS** system. The zone used in the vicinity of Fermilab site is the Illinois East Zone #1201 as given by the National Geodetic Survey (Stem, 1989).

The parameters of the projection are given by:

Grid Origin:

ϕ_0 = N 36° 40′ 00.00000″
λ_0 = W 88° 20′ 00.00000″

False coordinates applied at the origin are given by:

E_0 = 300000.0000 m ; False Easting at the grid origin
N_0 = 0.0000 m ; False Northing at the grid origin

Central meridian scale factor at the grid origin

F_0 = 0.9999750000

9. Acknowledgment

Thanks to all those who made this paper possible. The geodetic committee comprising of the author, Virgil Bocean, and George Wojcik defined and ratified the coordinate systems and projection parameters. Virgil Bocean computed the projection parameters which were also recomputed by the author for verification. Virgil Bocean also wrote Appendix A. Terry Sager, Stu Lakanen, John Greenwood, and the committee members reviewed this documentation. Helpful discussions with all the above mentioned are also gratefully acknowledged.

References

Leick, A., (1990). GPS Satellite Surveying, John Wiley & Sons, Inc., New York.

Schwarz, C. R., (1989). North American Datum of 1983, NOAA Professional Paper NOS 2, Ed. Charles R. Schwarz, National Geodetic Survey Rockville, MD.

Snyder, J. P., (1987). Map Projections - A Working Manual. U.S. Geological Survey Professional Paper 1395. U.S. Government Printing Office, Washington D.C.

Stem, J. E., (1989) . State Plane Coordinates System of 1983. NOAA Manual NOS NGS. National Geodetic Information Branch, NOAA, Rockville, MD.

The Fermilab Main Injector, Technical Design Handbook. Fermilab Main Injector Department, Fermi National Accelerator Laboratory, August 1994.

The FMI Collaboration Internal Communication. Fermi National Accelerator Laboratory, December 12, 1995.

Thompson, D.B., Mepham, M.P. and R.R. Steves, (1977). The Stereographic Double Projection. The Department of Surveying and Engineering, University of New Brunswick, Fredericton, N.B., Canada. Technical Report No. 46.

Vanicek, P., and Krakiwsky, E. J., (1982). Geodesy: The Concepts, North-Holland Publishing Company.

Appendix A

Spatial Definition of the FMI Plane

The objective of this section is to summarize the definition of the plane that contains the Fermilab Main Injector (FMI) with respect to the existing Tevatron and the local Earth parameters. The details of the FMI location are determined by the requirements for transfers of both protons and antiprotons into the Tevatron.

The FMI is situated southwest of the Tevatron Ring. The MI-60 straight section is parallel to the Tevatron F0 straight section, and has the FSCS Azimuth = 301^{o} 13’ 53.9”. The two beamlines are separated by 11.823 m horizontally. The design location of the MI-60 reference point is 13.222 m downstream from the F0 TeV point, and offset from the TeV straight section by 11.823 m (FMI Technical Design Handbook, 1994).

The FMI is designed to be a planar machine. The final vertical definition of the FMI plane begins by specifying that cell boundaries 522 and 620 are designed at the nominal orthometric height of 218.15314 m (DUSAF Datum), and allow the initial constraint on delta elevation between the FMI and TeV to vary (FMI Collaboration Internal Communication, 1995). Based on the latest (1995) as found elevation of the Tevatron F0 straight section quads string (220.48886 m DUSAF Datum), and the nominal FMI design elevation (218.15314 m DUSAF Datum), the FMI plane is located 2.33572 m below the Tevatron beam.

Additionally, to account for the relative tilt of the FMI and the Tevatron planes, and also for fully constraining the FMI plane in the geodetic space with respect to the local Earth parameters, it is further specified that the cell boundaries 522, 620, and 308 are placed at the same nominal design orthometric height of 218.15314 m (DUSAF Datum). The coordinates of the cell boundaries 308, 522, and 620 in the FMI Plane (and other coordinate systems) are given in Table A1.

Table A1. The Spatial Definition of the FMI Plane

	LATTICE COORDINATES (LTCS-XYZ)			LATTICE COORDINATES (LTCS-XYH)				ORTHO-		GEODETIC COORDINATES (GCS)		
POINT	EASTING	NORTHING	HEIGHT	EASTING	NORTHING	HEIGHT	dH	METRIC	GEOID	LATITUDE	LONGITUDE	ELLIPSOD
NAME	X	Y	Z DUSAF	X	Y	H DUSAF	NAVD88-DUSAF	HEIGHT H	HEIGHT N	N	W	HEIGHT h
								NAVD88	GEOID93			GRS80
	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	dd mm ss.ssssss	dd mm ss.ssssss	(m)
MI-60	30942.39545	29606.62571	218.15314	30942.39547	29606.62572	218.15109	-0.17308	217.97801	-32.77535	41 49 42.807203	88 15 48.863909	185.20266
CELL308	30465.32584	28800.10421	218.15314	30465.32582	28800.10421	218.15314	-0.17308	217.98006	-32.76746	41 49 31.863874	88 16 26.741957	185.21260
CELL522	31118.46148	29469.23840	218.15314	31118.46150	29469.23842	218.15314	-0.17308	217.98006	-32.77394	41 49 35.776572	88 15 46.562766	185.20612
CELL620	30739.17384	29699.22967	218.15314	30739.17387	29699.22967	218.15314	-0.17308	217.98006	-32.77632	41 49 49.243617	88 15 53.291223	185.20374
CFMI	30681.68595	29217.26838	218.15314	30681.68595	29217.26838	218.13488	-0.17308	217.96180	-32.77148	41 49 38.134927	88 16 08.184535	185.19032

Appendix B

Notes on Scale Compatibilities

S_c Distance between two points inverted from the coordinates.

S_a Actual distance between two points as measured in reality.

GCCS This is a Cartesian coordinate system, which does not involve the use of a projection. S_c and S_a will be equal.

FSCS:XYZ This is a Cartesian coordinate system. S_c and S_a will be equal.

FSCS:XYH This is a projection coordinate system that has been re-scaled by the appropriate scale factor F to be compatible with the 720 ft above DUSAF datum. Therefore a two dimensional distance S_a measured at the 720 ft height is related to S_c by the expression:

$$S_c = k \cdot S_a$$

where the projection scale factor k is equal to 1 at the origin.

Away from the 720 ft above DUSAF datum an additional scaling is required:

$$S_c = k \cdot \frac{F_{720}}{F_h} \cdot S_a$$

where F_h is the height scale factor at the measurement height, and F_{720} is the standard height scale factor (= 1.000029251309483).

LTCS:XYZ This is a Cartesian coordinate system. S_c and S_a will be equal.

LTCS:XYH This is a projection coordinate system that has been

re-scaled by the appropriate scale factor F to be compatible with the 715.664 ft above DUSAF datum. In a similar definition to that used for the **FSCS:XYH**, the relationship between the two dimensional distance S_c and S_a is:

$$S_c = k \cdot \frac{F_{CFMI}}{F_h} \cdot S_a$$

where k is the projection scale factor, equal to 1 at the origin CFMI. F_h is the height scale factor at the measurement height, and F_{CFMI} is the standard height scale factor (= 1.000029046120306).

LGS This is a Cartesian coordinate system. S_c and S_a will be equal.

OMPS This is a projection coordinate system that has been re-scaled by the appropriate scale factor F_0 (= 1.000029251309483) to be compatible with the 720 ft above DUSAF datum.

In a similar definition to that used for the **FSCS:XYH**, the relationship between the two dimensional distance S_c and S_a is:

$$S_c = k \cdot \frac{F_{720}}{F_h} \cdot S_a$$

where k is the projection scale factor, equal to 1 along the center line of the projection (azimuth α); F_h is the height scale factor at the measurement height; and F_{720} is the standard height scale factor at the DUSAF datum.

ISPS This is a projection coordinate system that has not been re-scaled by to any datum.

The relationship between the two dimensional distance S_c and S_a is:

$$S_c = k \cdot \frac{1}{F_h} \cdot S_a$$

where k is the projection scale factor, equal to 0.999975 along the central meridian line of the projection (azimuth α); F_h is the height scale factor at the measurement height, equal to 1 on the ellipsoid.

Appendix C

Notes on Precision and Accuracy

The precision of the conversion from one coordinate system to another has been specified by the SAG group to be $1x10^{-7}$ m. Therefore in converting from one system to another and back again, the coordinates will be compatible to this level.

In terms of accuracy, it should be noted that a coordinate conversion cannot compensate for the inaccuracies in the original data. Therefore in converting to the **ISPS**, for example, the compatibility with other data can only be as good as the survey that defined the geodetic coordinates of the point A0.

Appendix D

Double Stereographic Projection

Traditional geodetic computations are carried out on the surface of a biaxial ellipsoid (ellipsoid of revolution), which is the mathematical figure that is the most convenient representation of the size and shape of the earth. To perform the same computations on a plane, it is necessary to map the ellipsoidal information (points, angles, lines, etc.) on a plane mapping surface. A convenient mapping for geodetic purposes is a conformal mapping in which ellipsoidal angles are preserved on the mapping plane.

The Stereographic projection of an ellipsoid of revolution can be approached by a double projection (hence double Stereographic projection), in which the biaxial ellipsoid is conformally mapped to a sphere, which is then "stereographically" projected to a plane.

The Stereographic projection of a sphere to a plane has the following properties (Thompson, et. al., 1977):

(i) It is a perspective projection, the features being projected from the sphere onto the plane from a common point of perspective. This perspective center lays on the sphere, and is the antipodal point of the point at which the plane is tangent to the sphere.
(ii) It is an azimuthal projection, characterized by the fact that the direction, or azimuth, from the center of the projection to every other point
on the map is shown correctly.
(iii) It is a conformal projection, the relative local angles about every point on the map are shown correctly (angles not distorted).
(iv) Scale increases away radially from the origin (center) of the projection.
(v) Great circles are projected as circles.

The Stereographic projection of a sphere to a plane is the only true perspective projection of any kind that is also conformal. There is no mapping of an ellipsoid to a plane that possesses all the characteristics of the Stereographic mapping of a sphere to a plane. For the conformal mapping of the biaxial ellipsoid on a plane, the ellipsoidal data is first mapped conformally on to a sphere. Then, a second conformal mapping of the spherical data to the plane completes the double Stereographic projection. The result is a conformal mapping of ellipsoidal data on a plane, since the two mappings are conformal.

The Stereographic projection is the most widely used azimuthal projection, mainly used for portraying large areas of similar extent in all directions. The Stereographic projection is very suitable when areas of interest are circularly shaped, like the FMI area. Since all the surveying activities will occur along the path of the ring, choosing the origin of the projection at the centroid of the FMI Plane has as a result that the scale along the tunnel will be standardized to the same scale factor. The differences created by the fact that the FMI shape is not perfectly circular is very insignificant.

Appendix E

Lattice Program Documentation and Source Codes

ERRATA
LATTICE PROGRAM DOCUMENTATION

On page 3,

Geodetice Cartesian Coordinate System

should read

Geodetic Cartesian Coordinate System

On page 6,

$$\mathbf{X_{GC}} = \begin{matrix} X_{GC} \\ Y_{GC} \\ Y_{GC} \end{matrix} \quad \text{should read} \quad \mathbf{X_{GC}} = \begin{matrix} X_{GC} \\ Y_{GC} \\ Z_{GC} \end{matrix}$$

$$\mathbf{X_{LG}} = \begin{matrix} X_{LG} \\ Y_{LG} \\ Y_{LG} \end{matrix} \quad \text{should read} \quad \mathbf{X_{LG}} = \begin{matrix} X_{LG} \\ Y_{LG} \\ Z_{LG} \end{matrix}$$

On page 8,

$$\mathbf{X_{FXYZ}} = \begin{matrix} X_{FXYZ} \\ Y_{FXYZ} \\ Y_{FXYZ} \end{matrix} \quad \text{should read} \quad \mathbf{X_{FXYZ}} = \begin{matrix} X_{FXYZ} \\ Y_{FXYZ} \\ Z_{FXYZ} \end{matrix}$$

$$\mathbf{X_{LG}} = \begin{matrix} X_{LG} \\ Y_{LG} \\ Y_{LG} \end{matrix} \quad \text{should read} \quad \mathbf{X_{LG}} = \begin{matrix} X_{LG} \\ Y_{LG} \\ Z_{LG} \end{matrix}$$

On page 8,

a = Geodetic Azimuth of the Fermilab (XYZ) Origin

should read

α = Geodetic Azimuth of the Fermilab (XYZ) Origin

On page 12,

α = Geodetic Azimuth of the Fermilab (XYZ) Origin