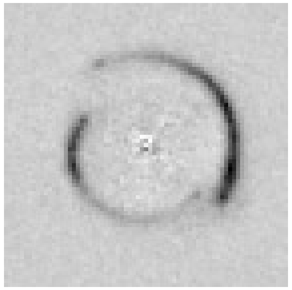
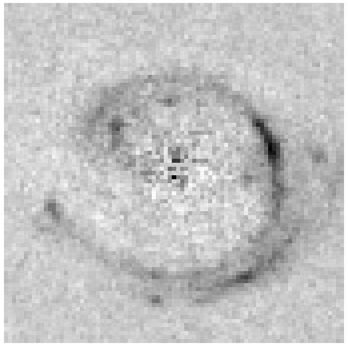




Modeling resolved gravitational lenses

Randall Wayth

Harvard-Smithsonian Center for Astrophysics

Outline

- An overview of the problem
- Lens analysis algorithms and software
- Examples
- Statistics for resolved images
- Computational and practical issues

What exactly is the image?

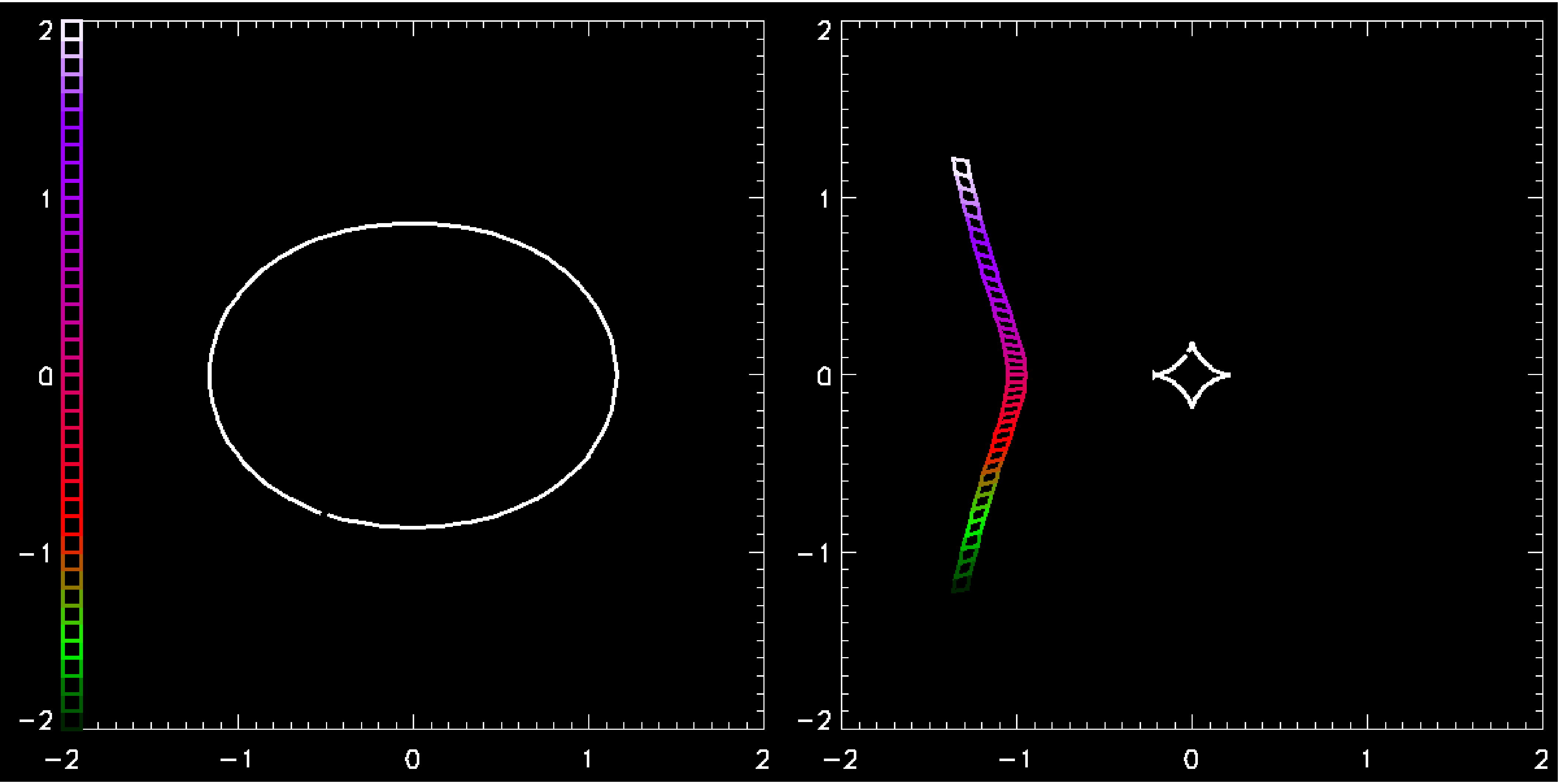
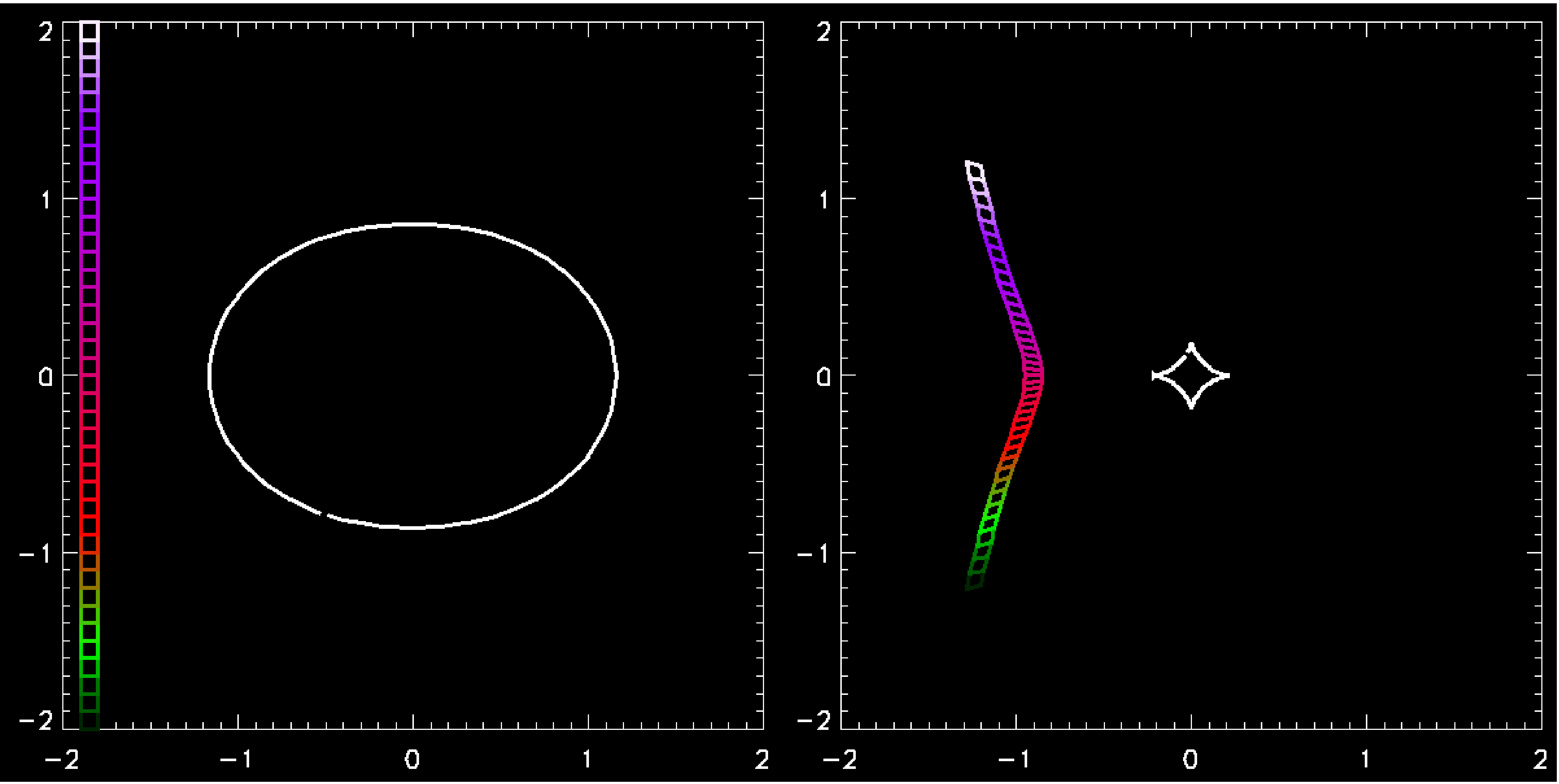


image plane

source plane



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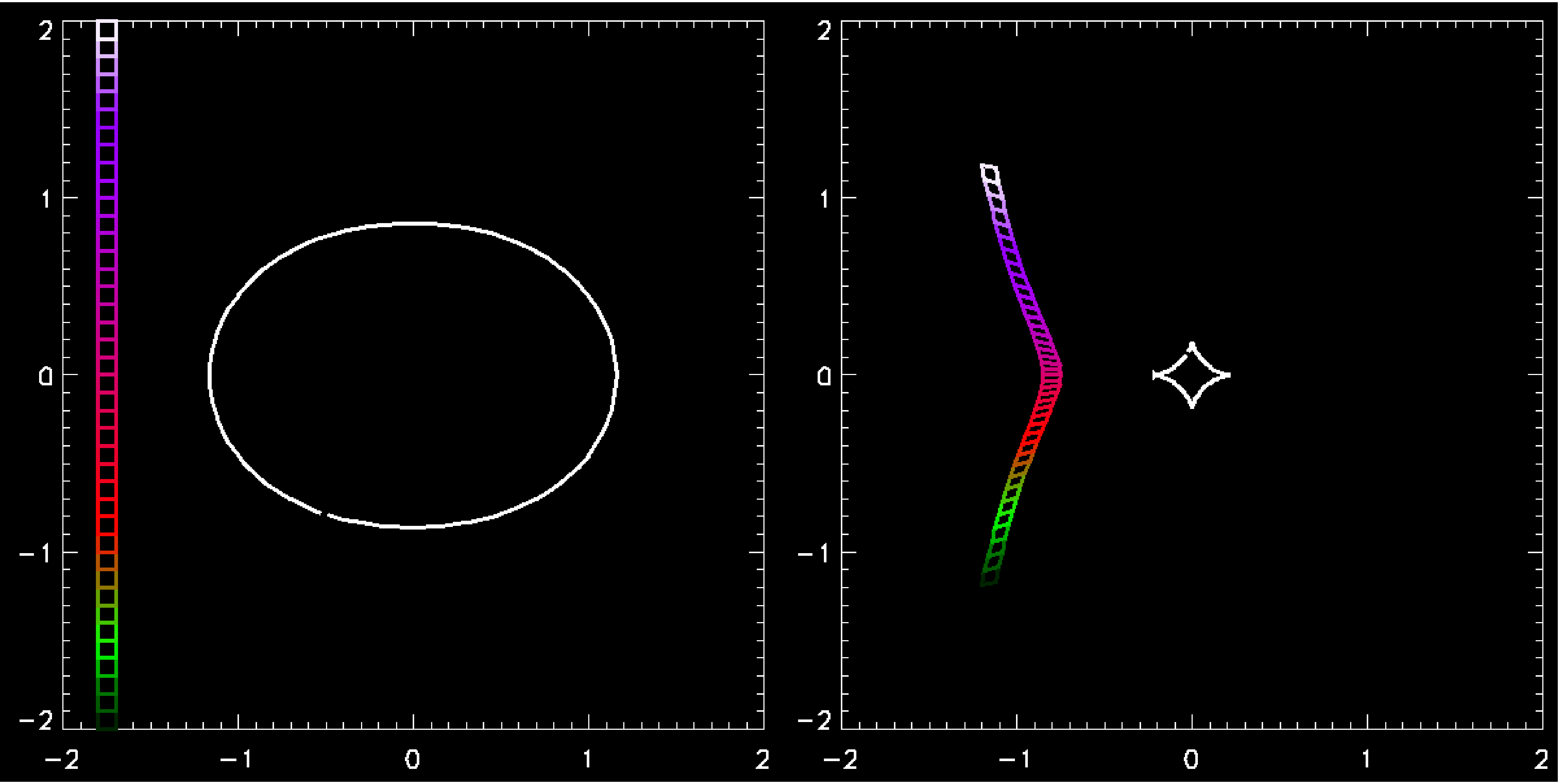

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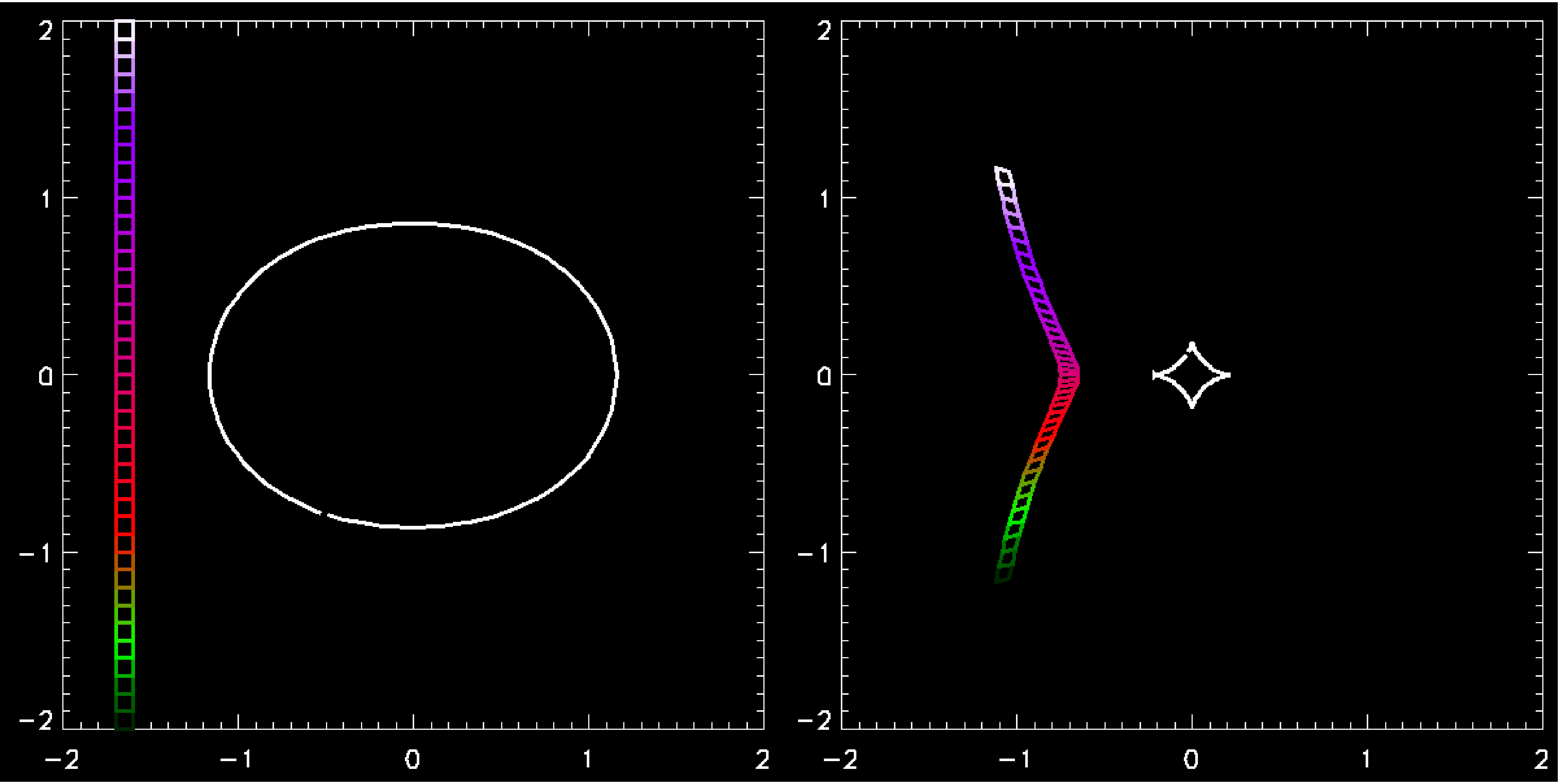

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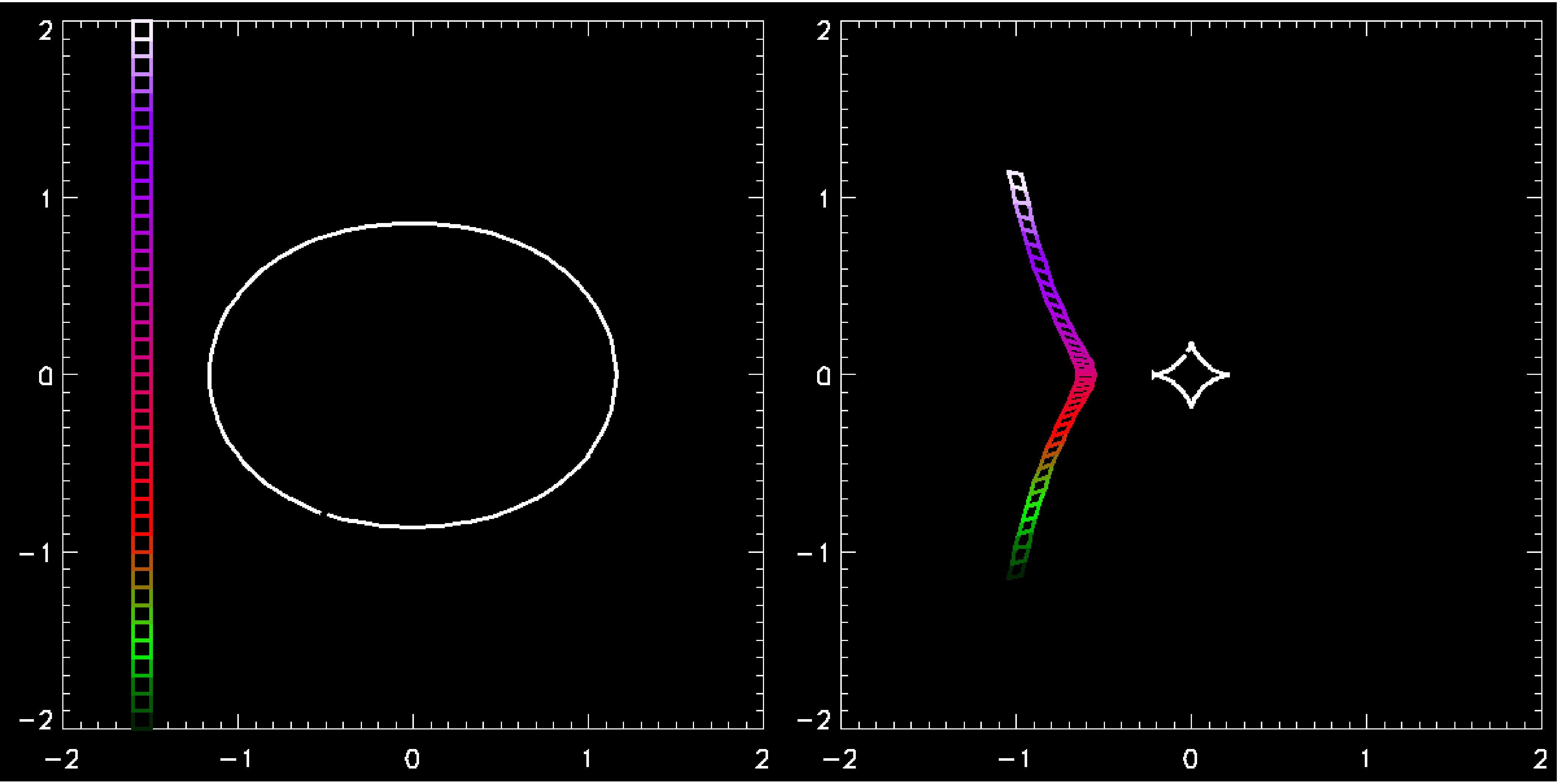
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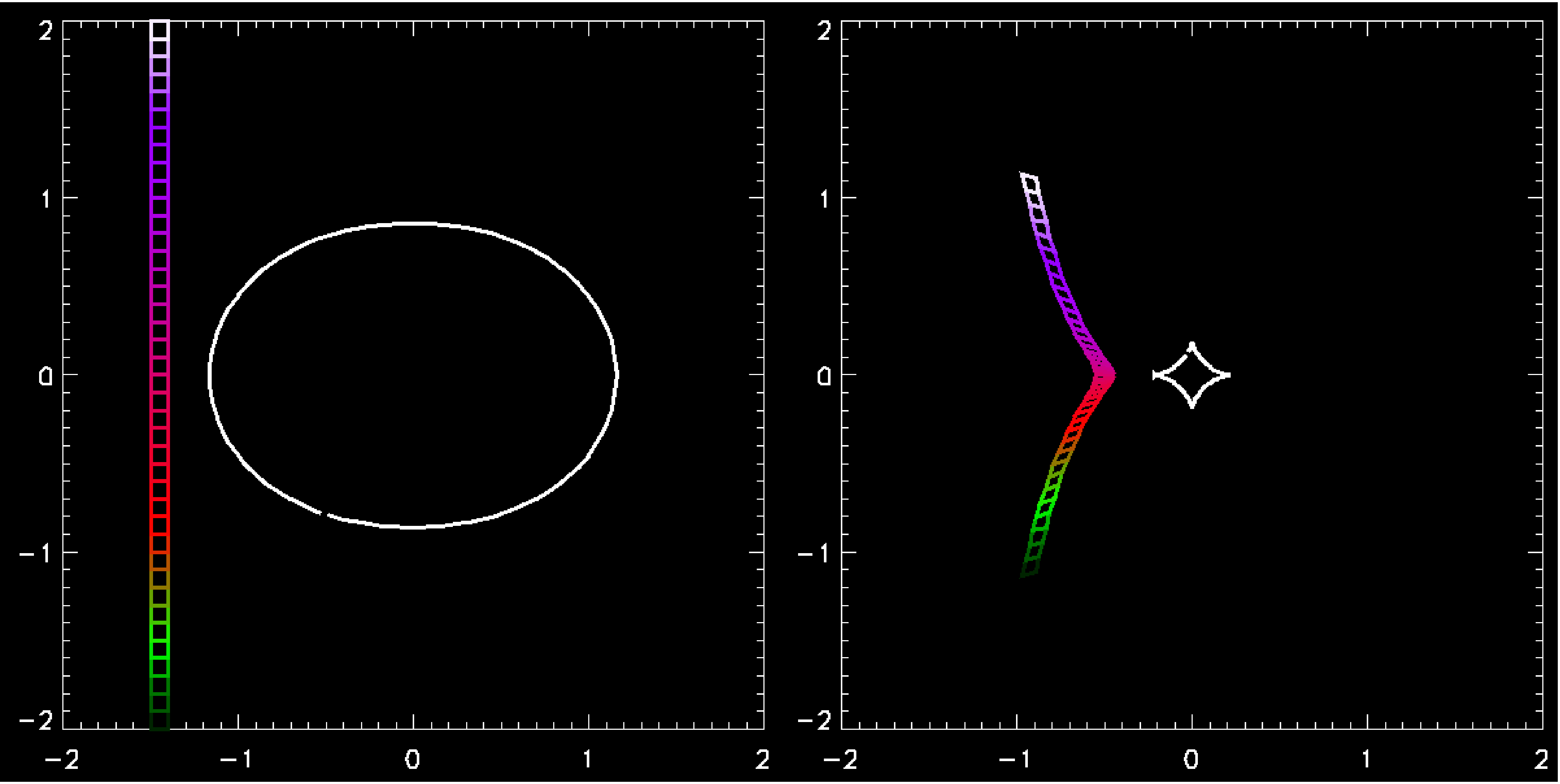

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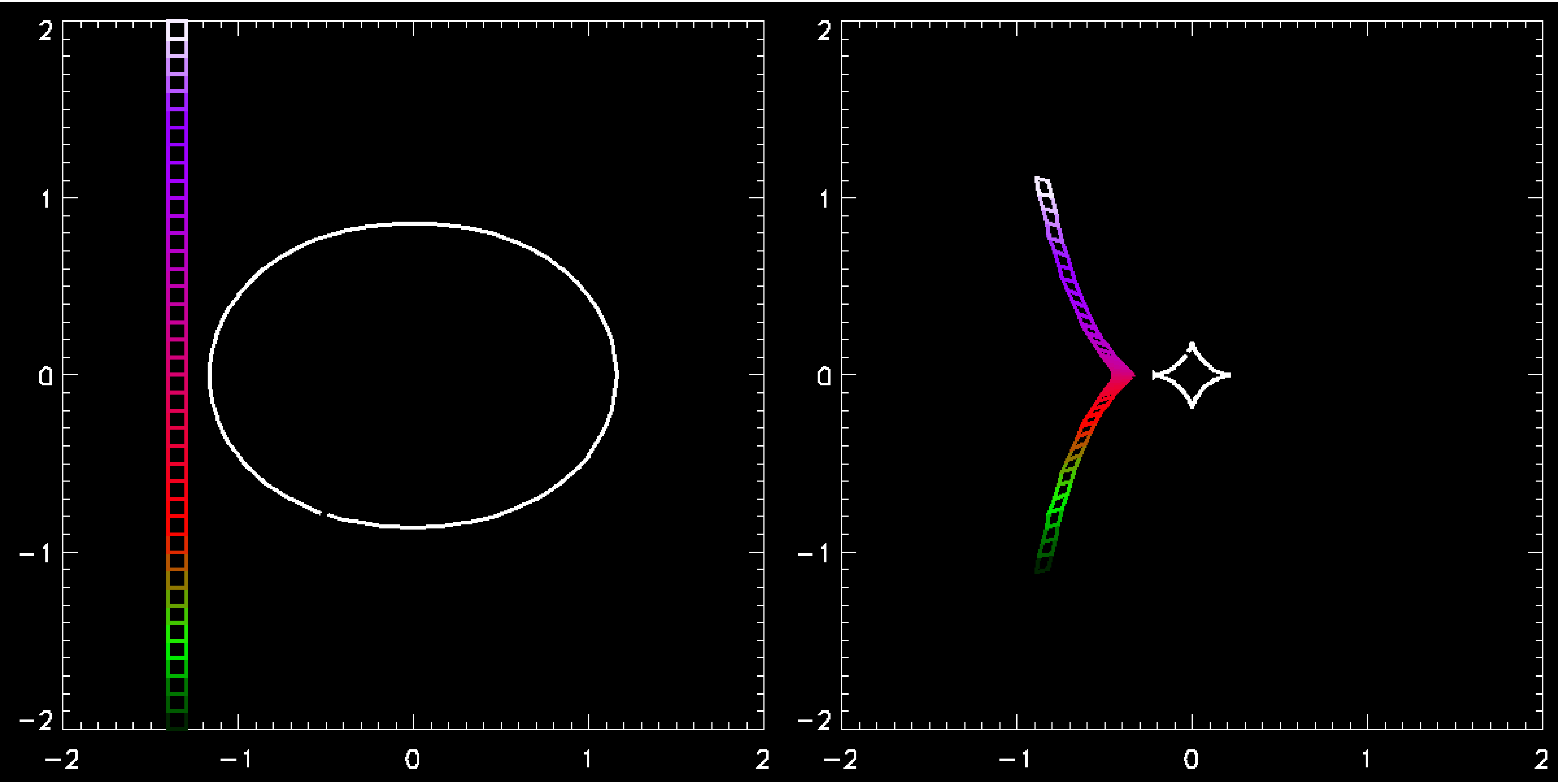
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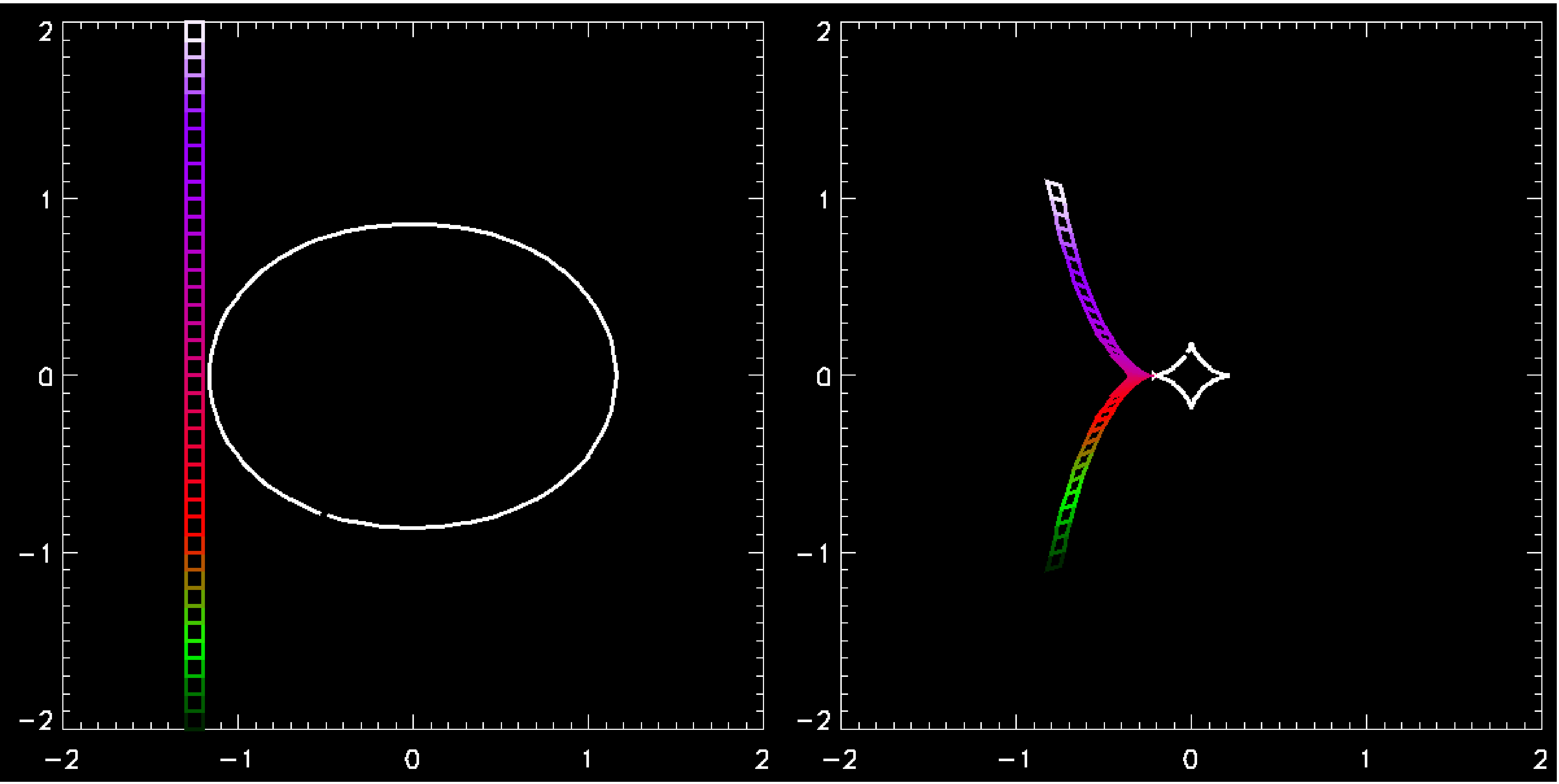

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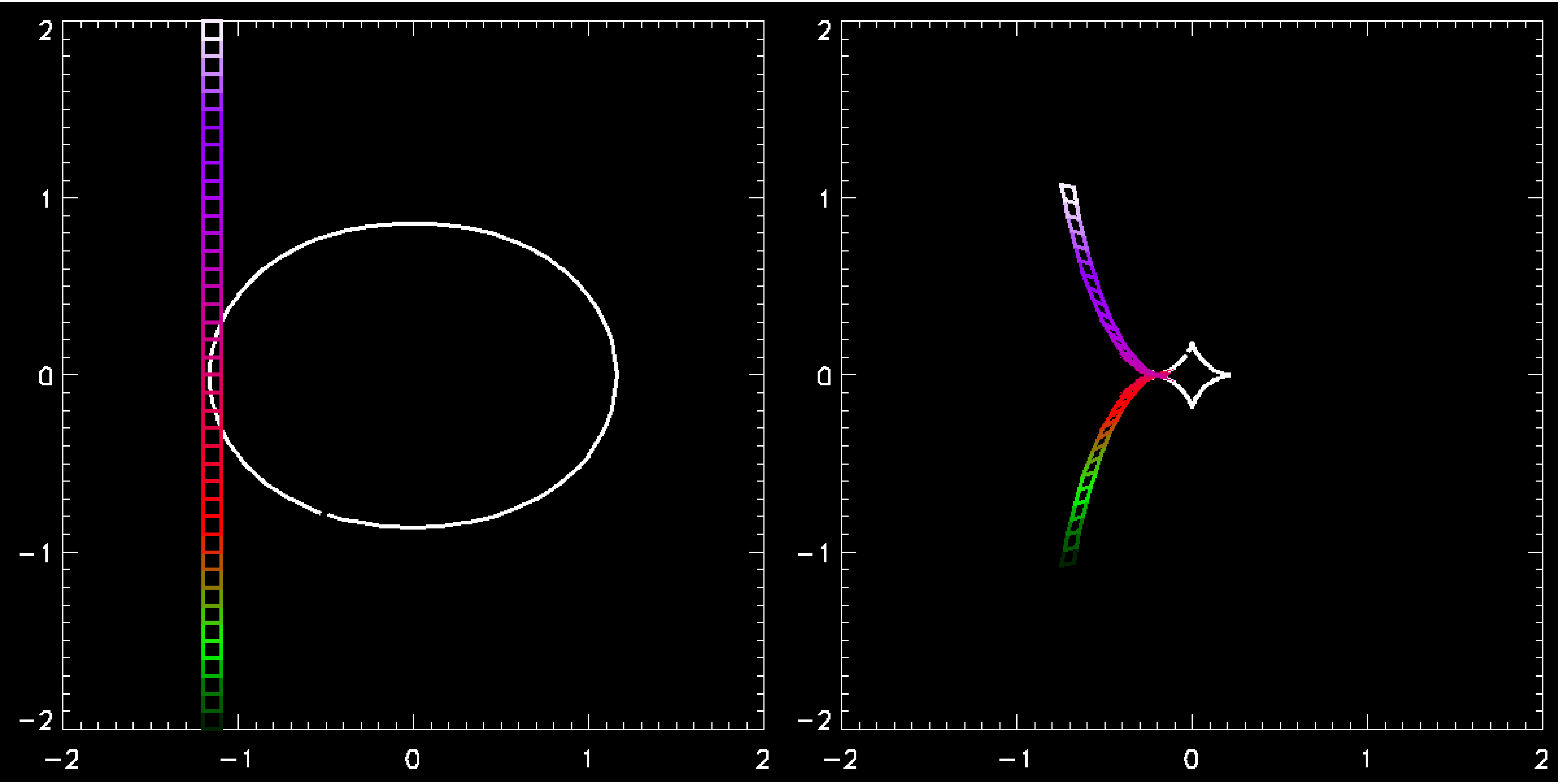
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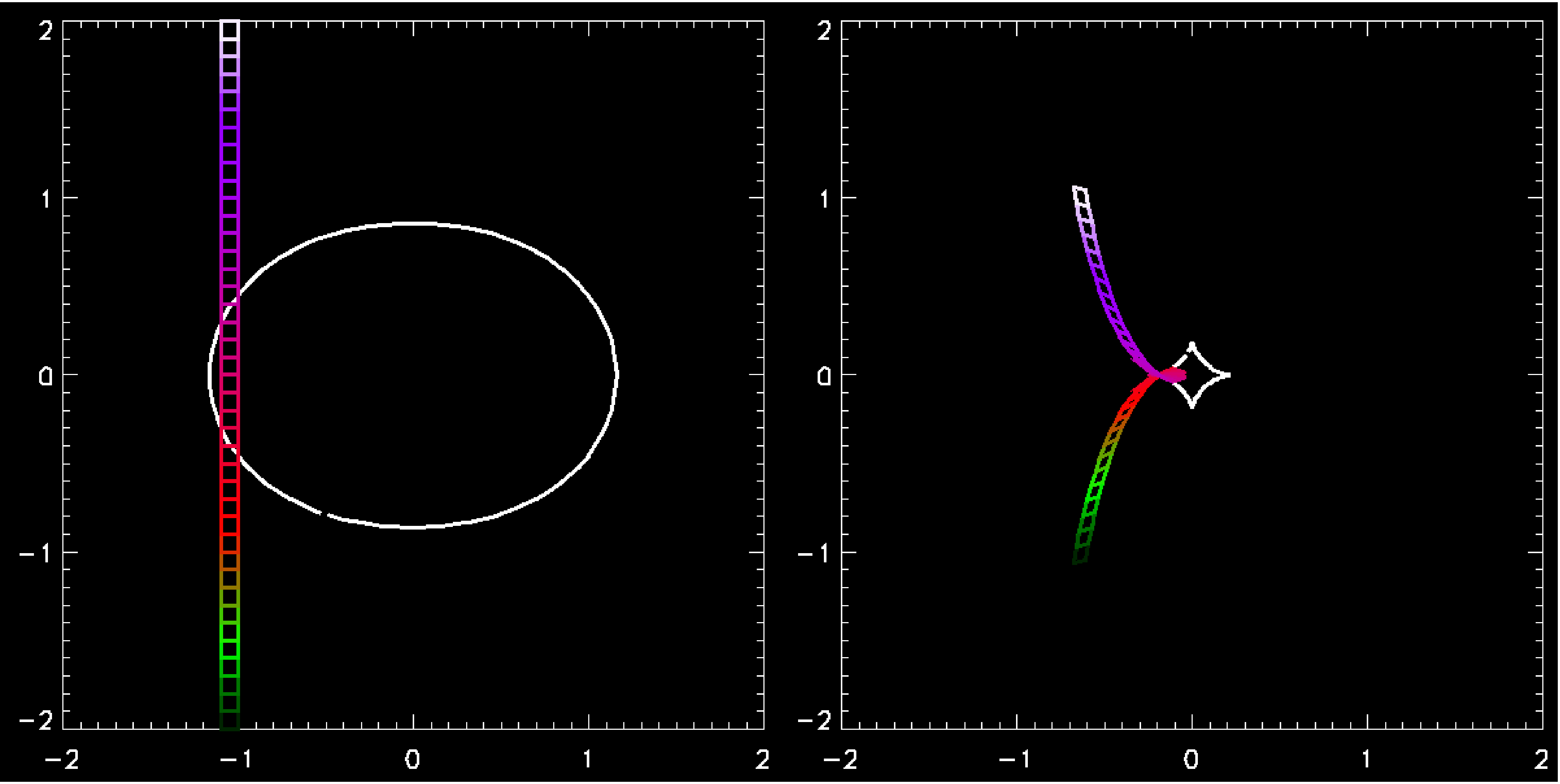
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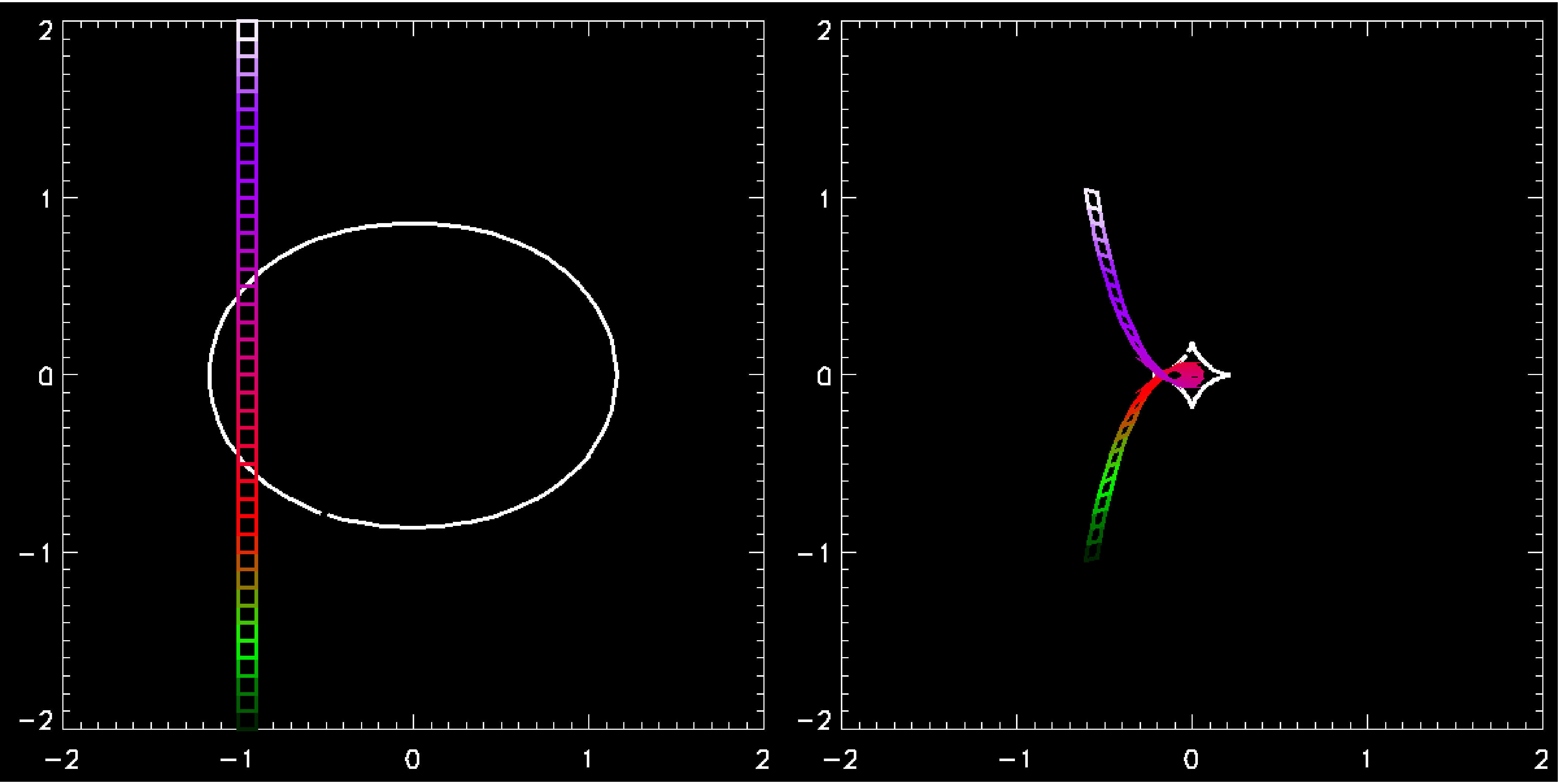

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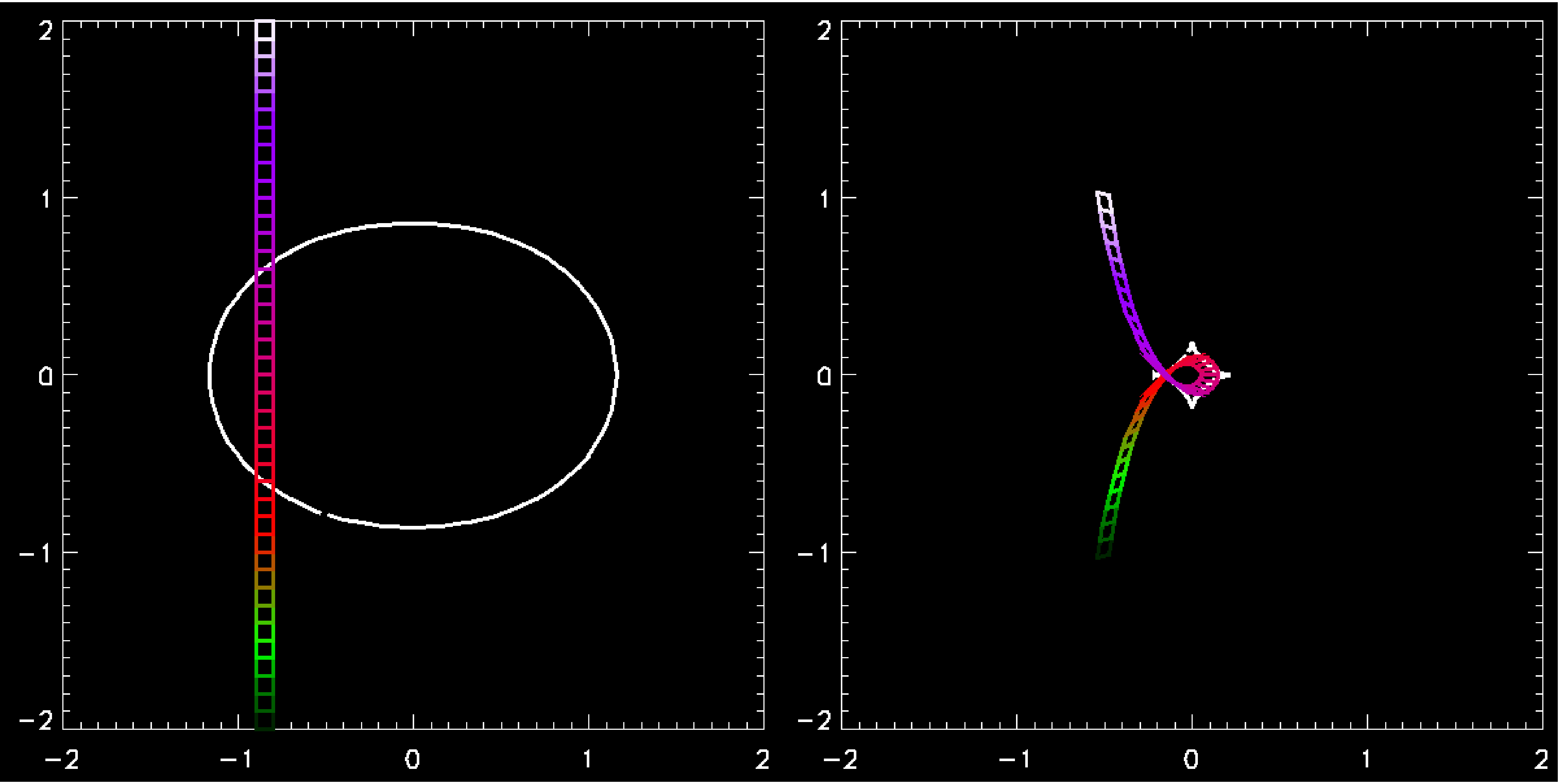

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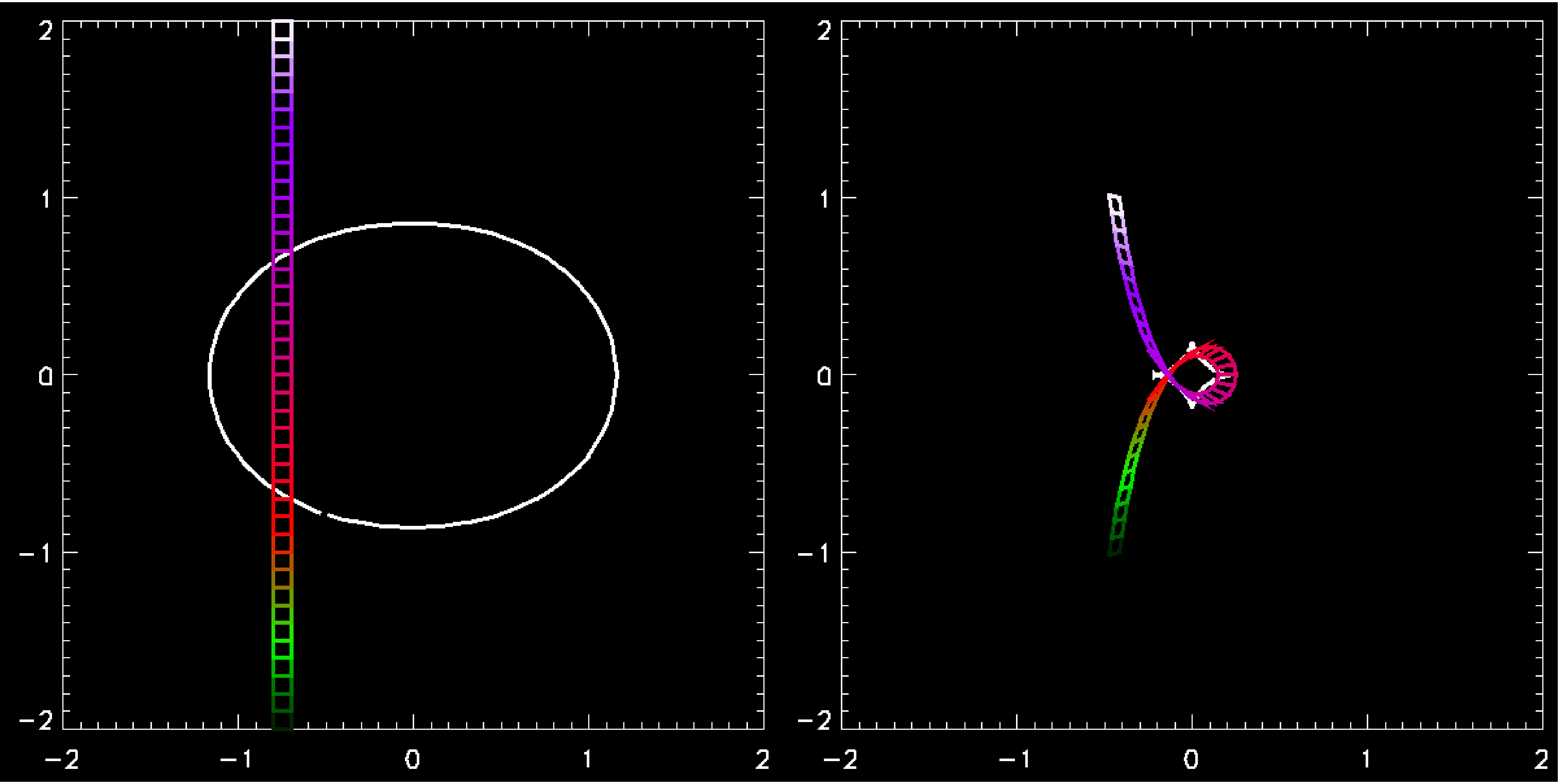
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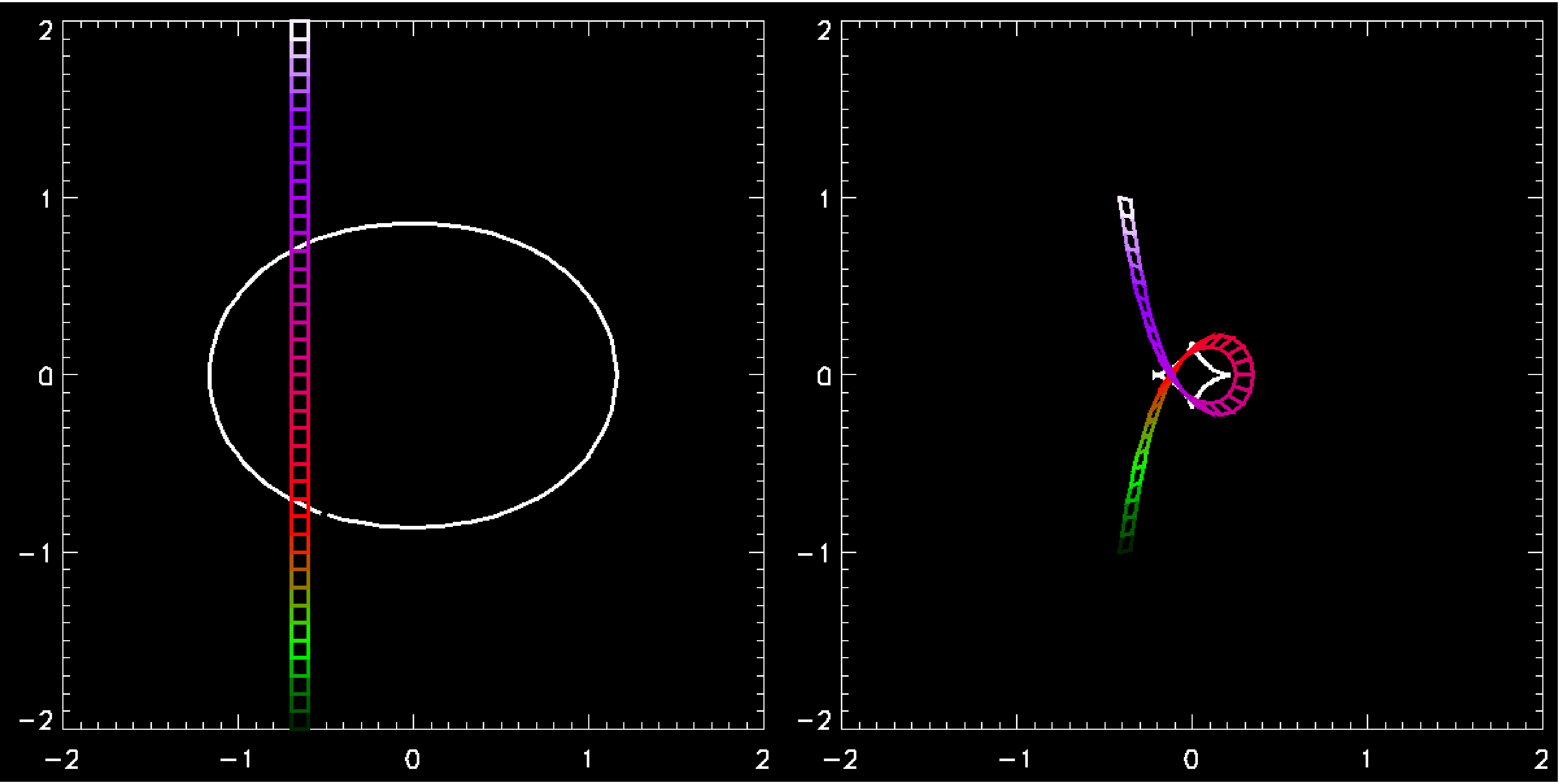

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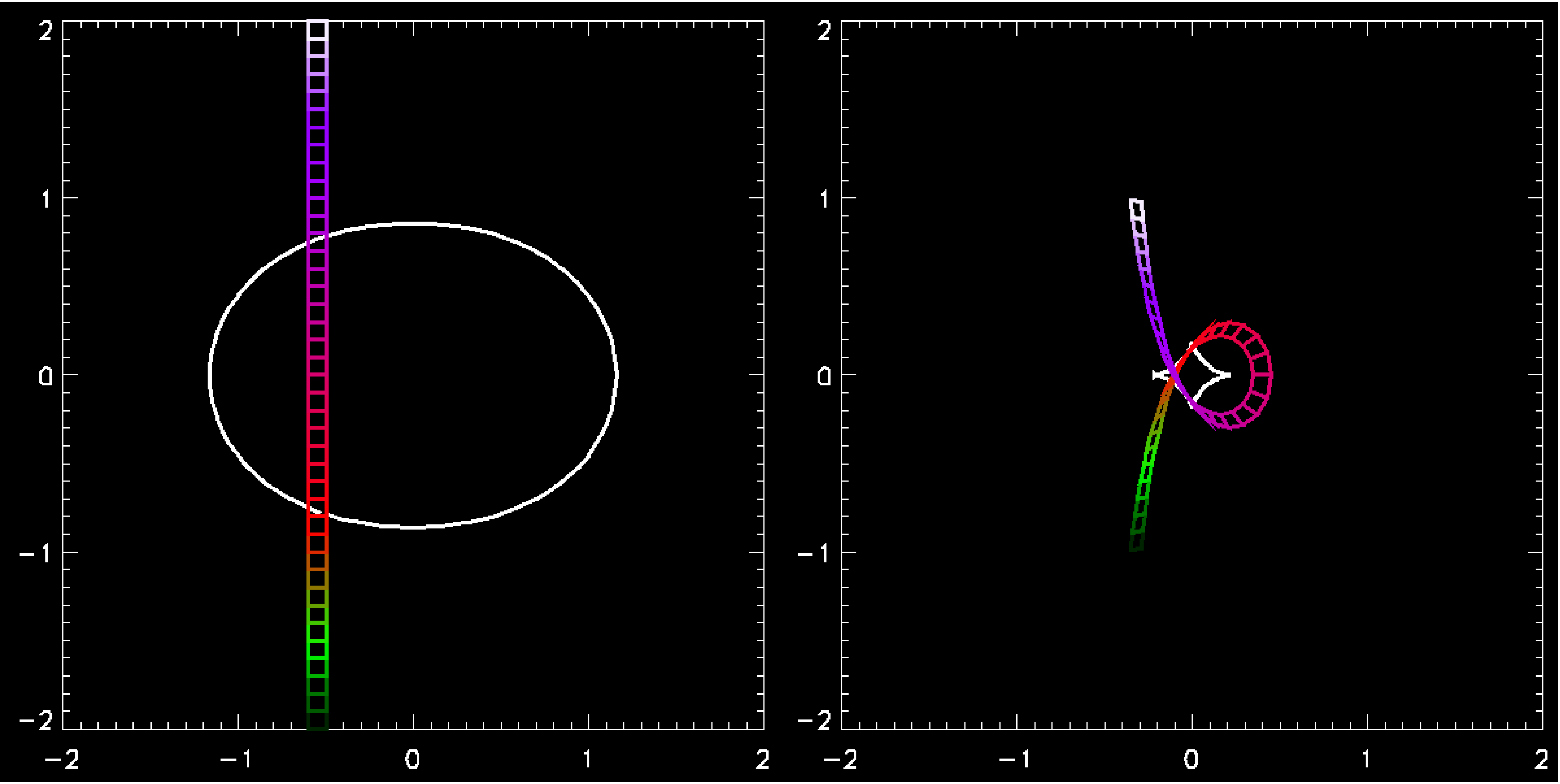

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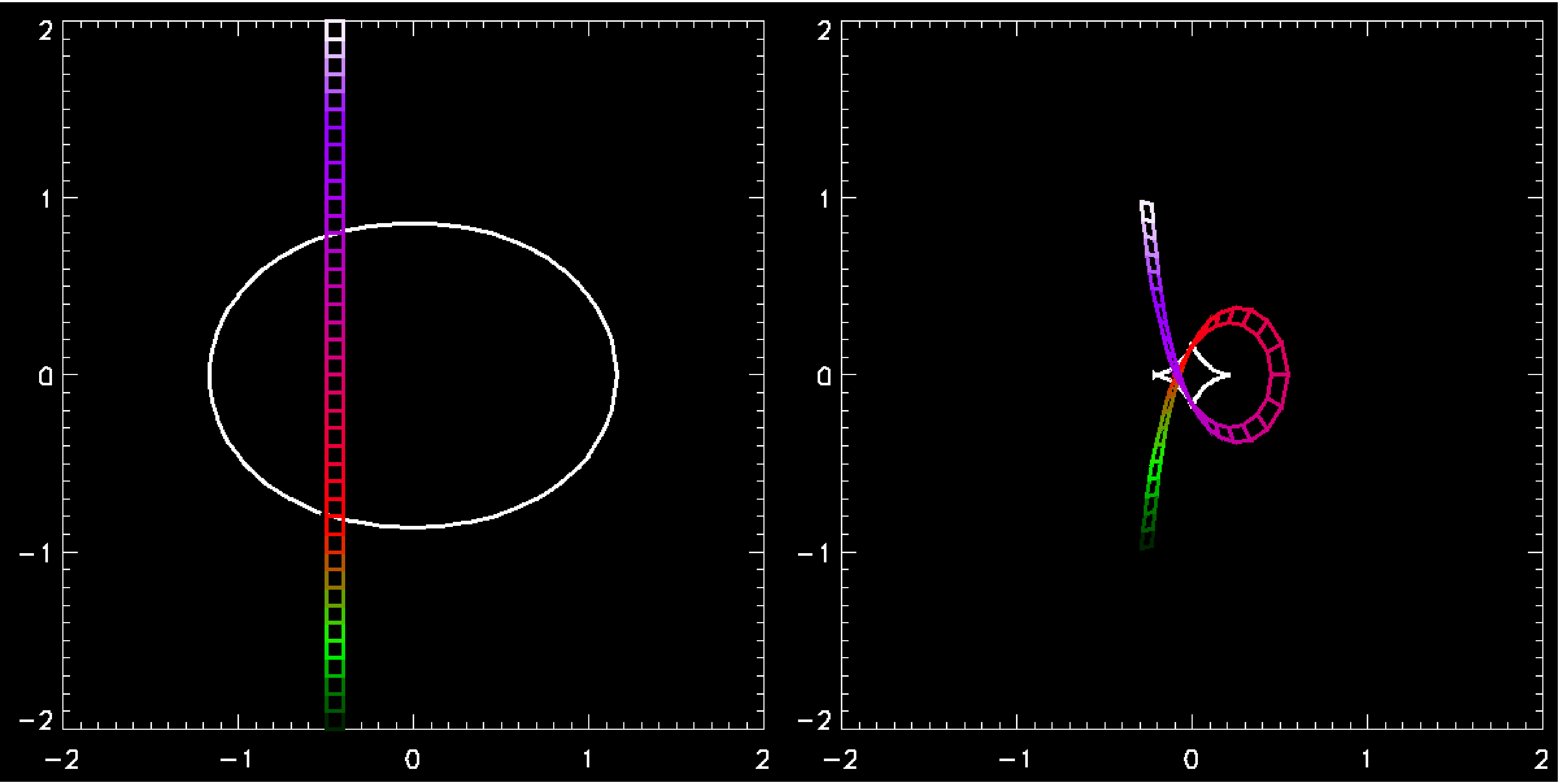
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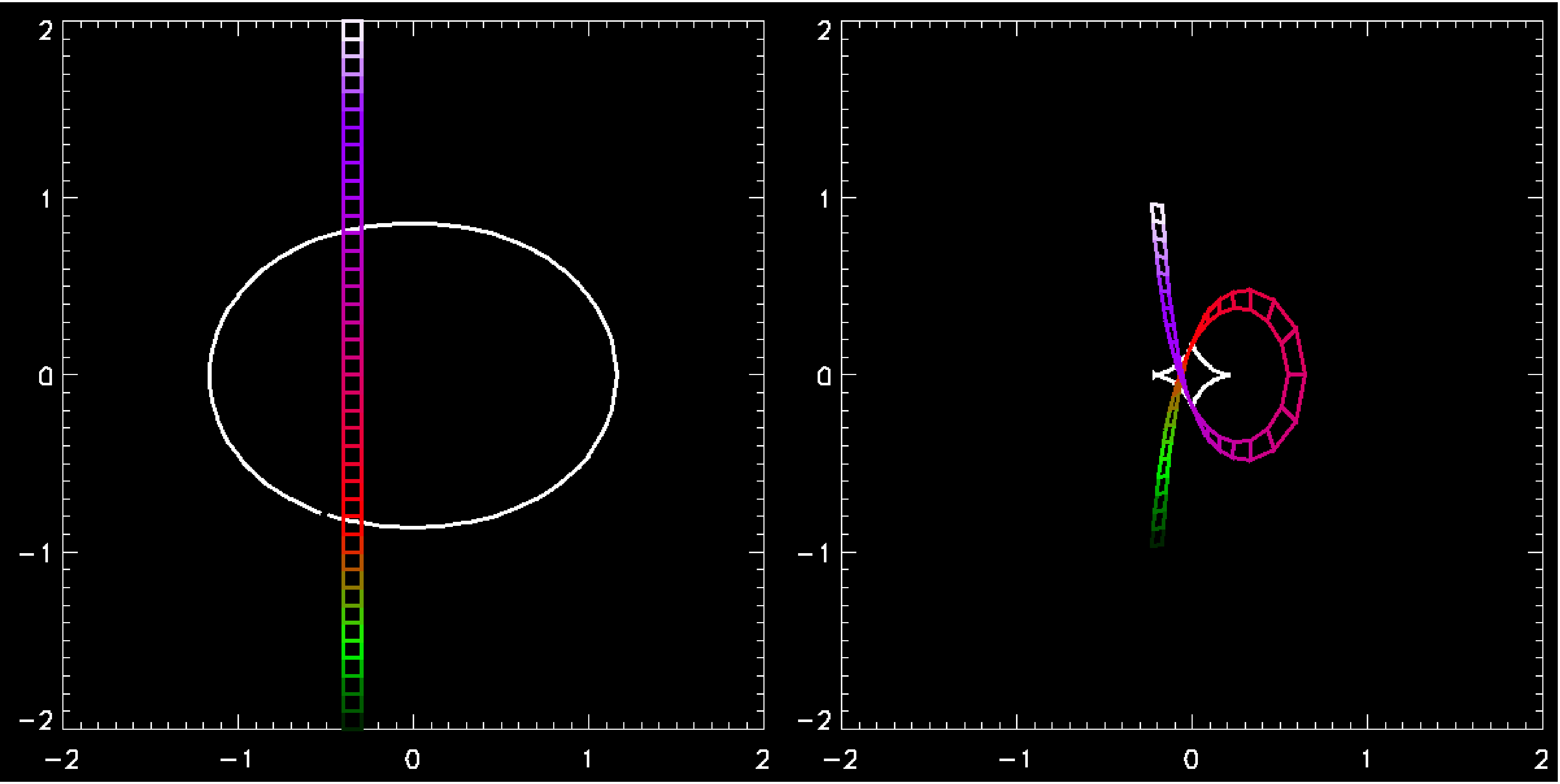

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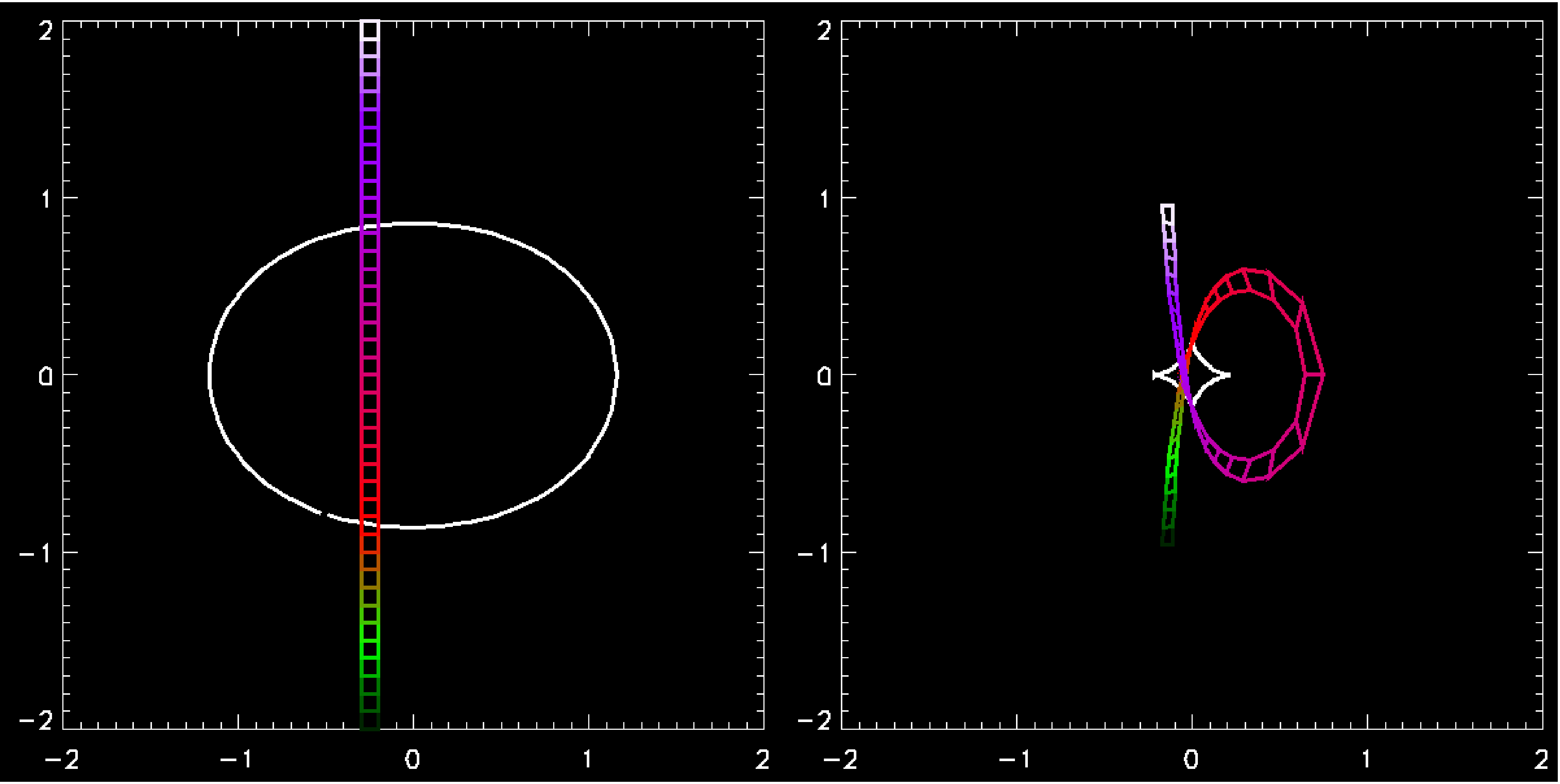

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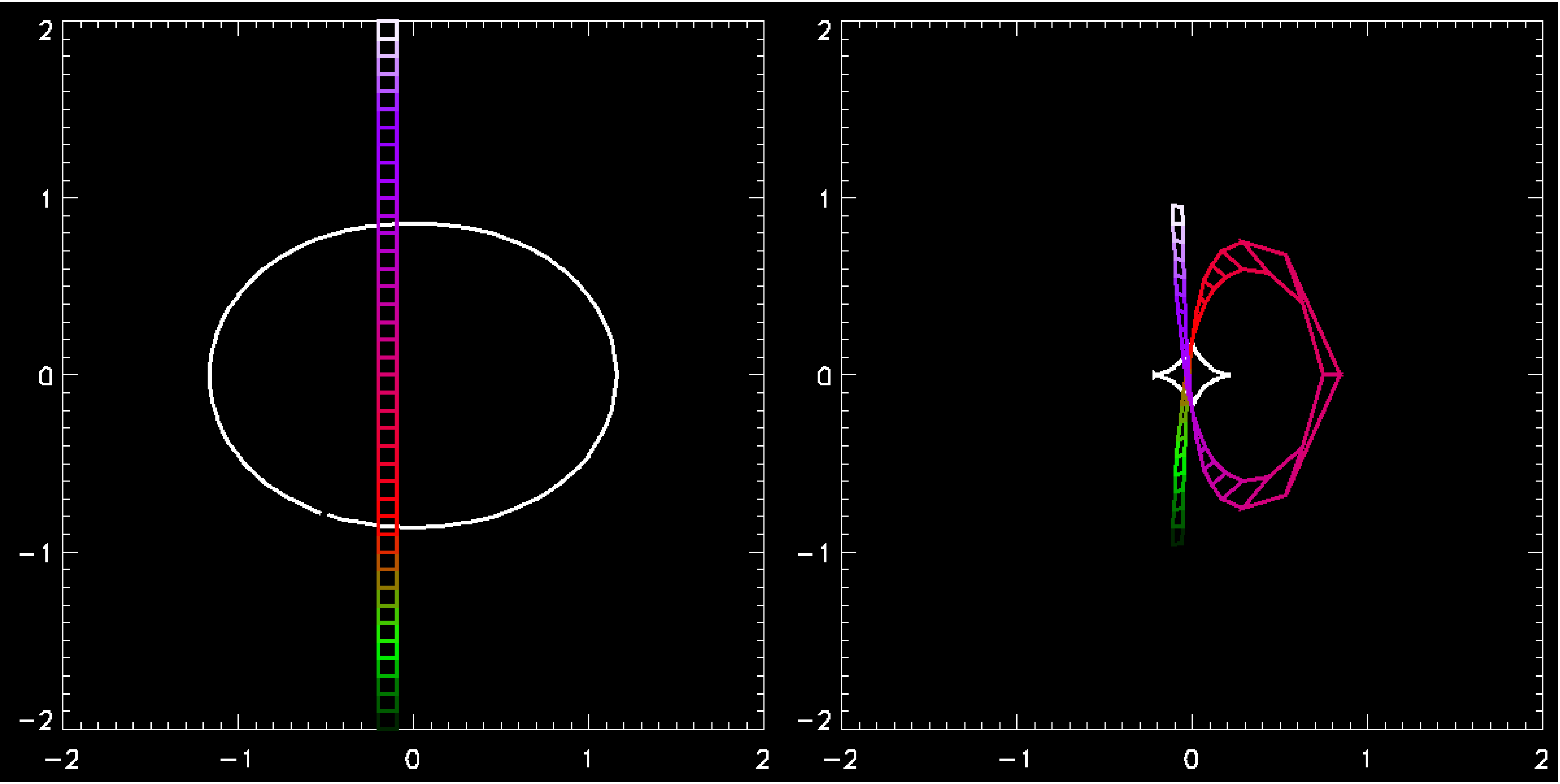

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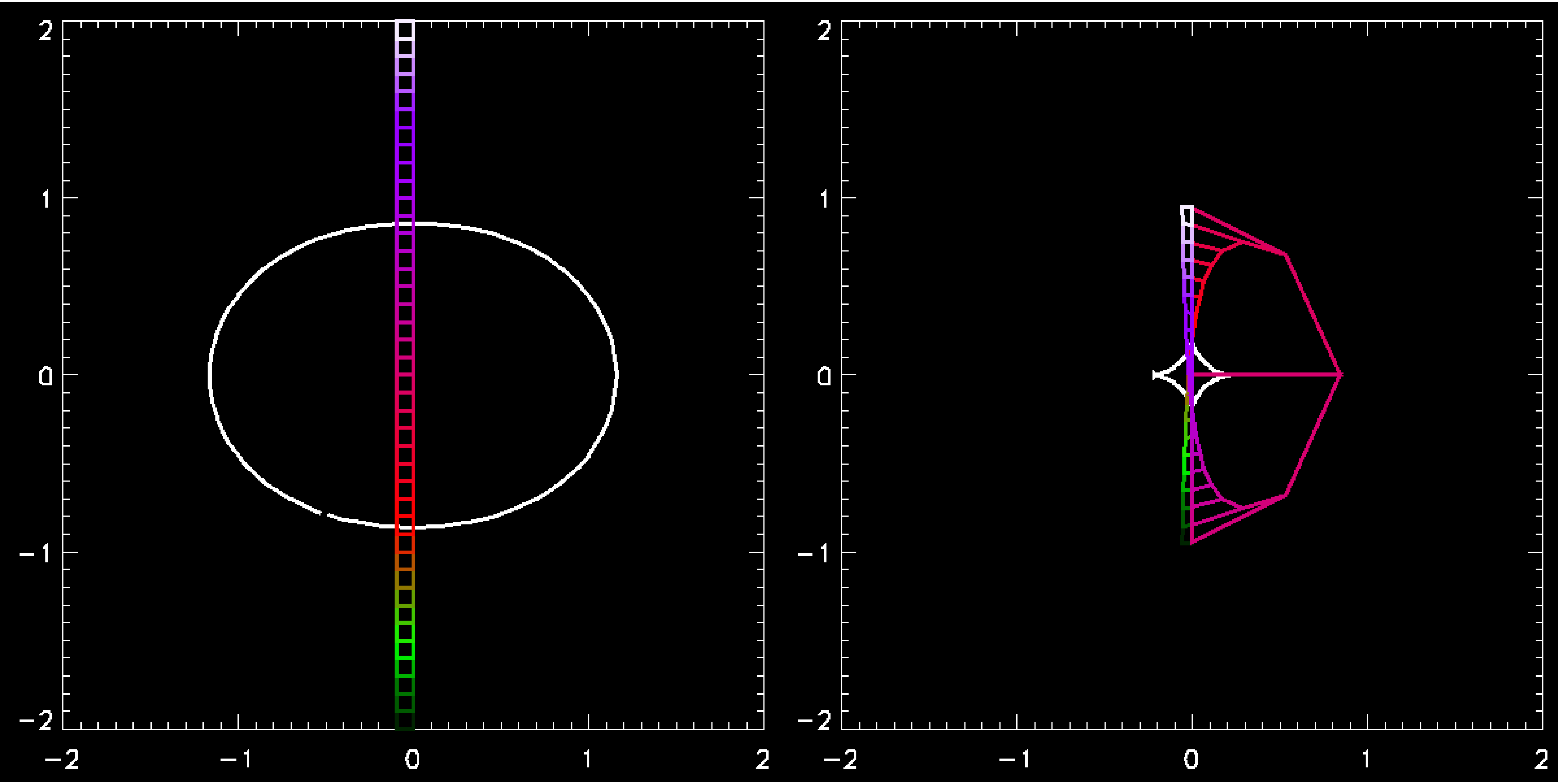

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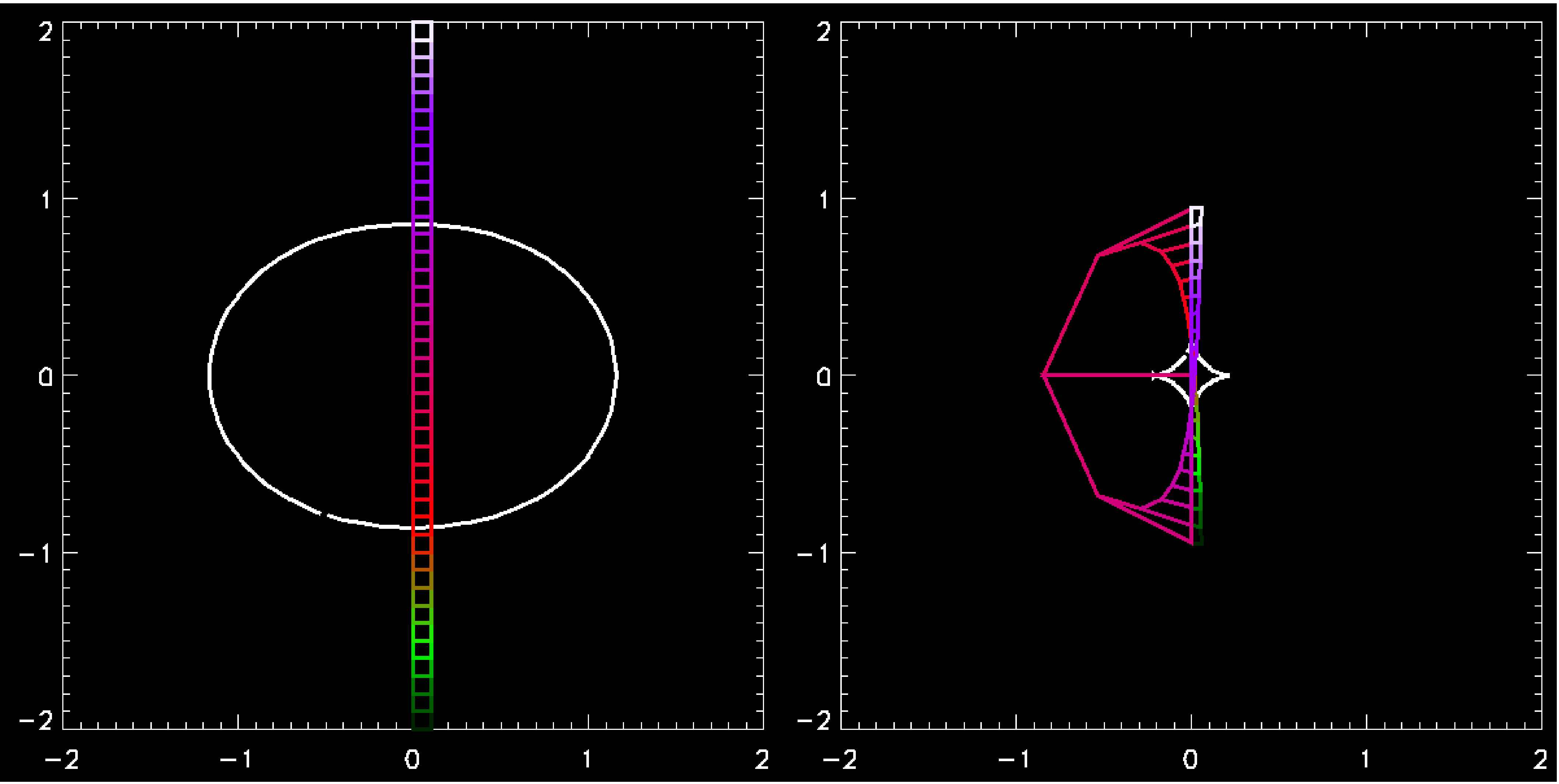
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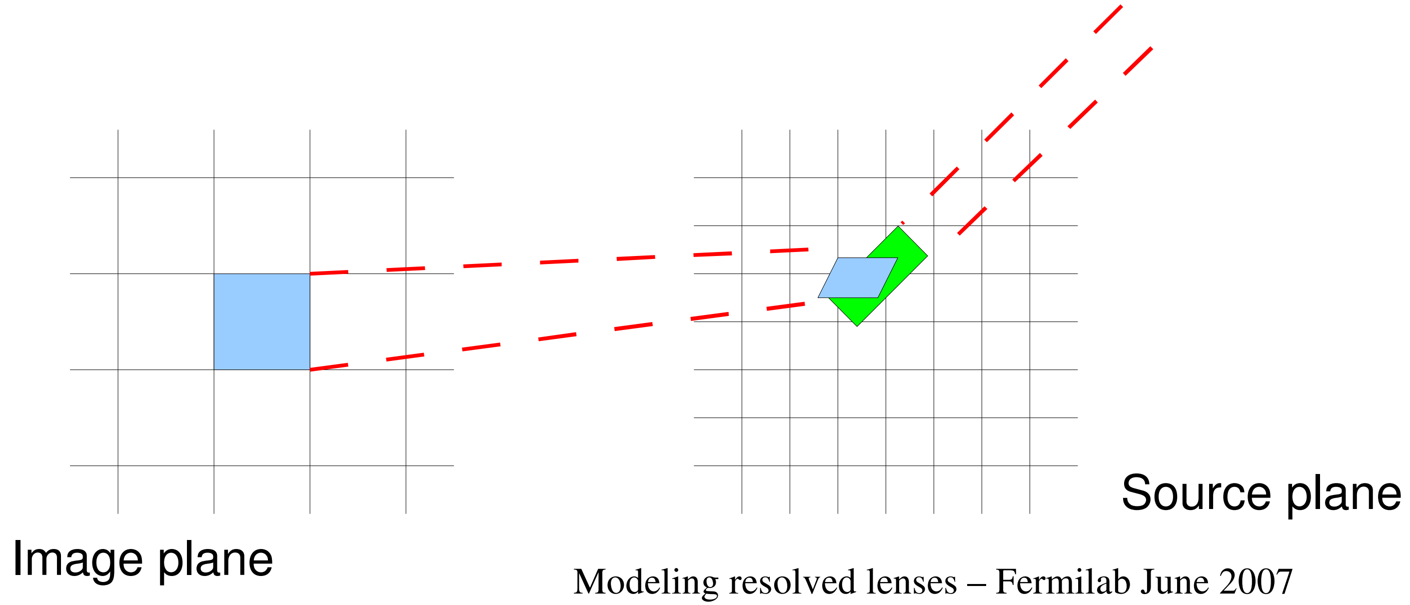
Overview of the problem

- Each pixel in the image has the average surface brightness of a patch of the source plane (ignoring PSF)
- For pixelized source, each image pixel's projection will overlap several neighboring source plane pixels. Image pixel is a weighted sum of source pixels.



Overview of the problem

- This is a one-to-many mapping problem
- If we unwrap the image and source planes into vectors, we can write

$$y = Rx + n$$

y = measured pixels

x = unknown source pixels

R = "response" or mapping matrix

n = noise

- This is a textbook inverse problem. (See Numerical Recipes chapters 18,15)
- **R** is an $N_{imgpixels}$ x $N_{srcpixels}$ matrix. Sparseness depends on lens model, PSF.

Overview of the problem

- Aim: *for a fixed lens model*, get best estimator for **x**., i.e. make a source model which best reproduces the image.
- In general, there are two approaches:
  - linear inversion: solve a big matrix equation to get **x** in one step
  - iterative methods: use difference between model and data to improve model in successive steps (must use iterative when matrix is too large)
- With lensing, we also (mostly?) want the mass model

Modeling algorithms

- Desirable qualities: stable with noise, non-parametric source, no high frequency components in source
- Inverse/deconvolution problems can have an infinite number of statistically acceptable solutions- how to choose the “best” one?
  - apply “regularization”, which picks a solution from the many according to some rule.
- Regularization is just an *a priori* constraint. e.g.:
  - positivity of source
  - “smoothness” of source

Review of resolved lens analysis tools

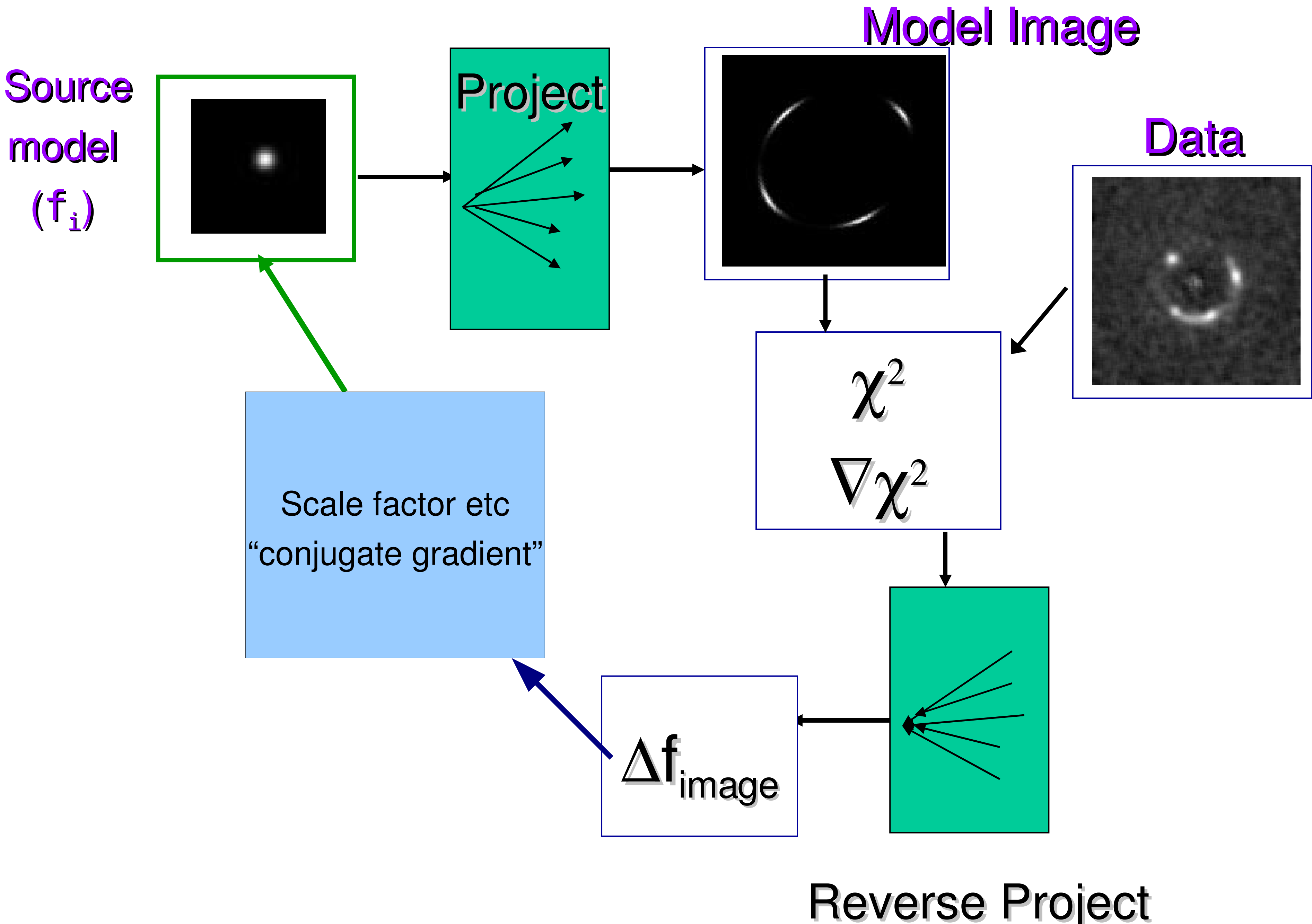
- *Ring Cycle* (Kayser & Schramm 1988, Kochanek et al 1989)
  - Parametric source, no PSF
- *LensCLEAN* & variants (Kochanek & Narayan 1992, Ellithorpe et al 1996, Wucknitz et al 2004)
  - Designed for interferometer data. Build a model of the source using CLEAN like algorithm
  - Practical only for analytic lens models
- *Semi-linear inversion* & variants (Warren & Dye 2003, Treu & Koopmans 2004, Brewer & Lewis 2005)
  - Image deconvolution through linear inversion. Non-parametric source
  - Feasibilty for broad (non HST) PSF?

... and Lensview

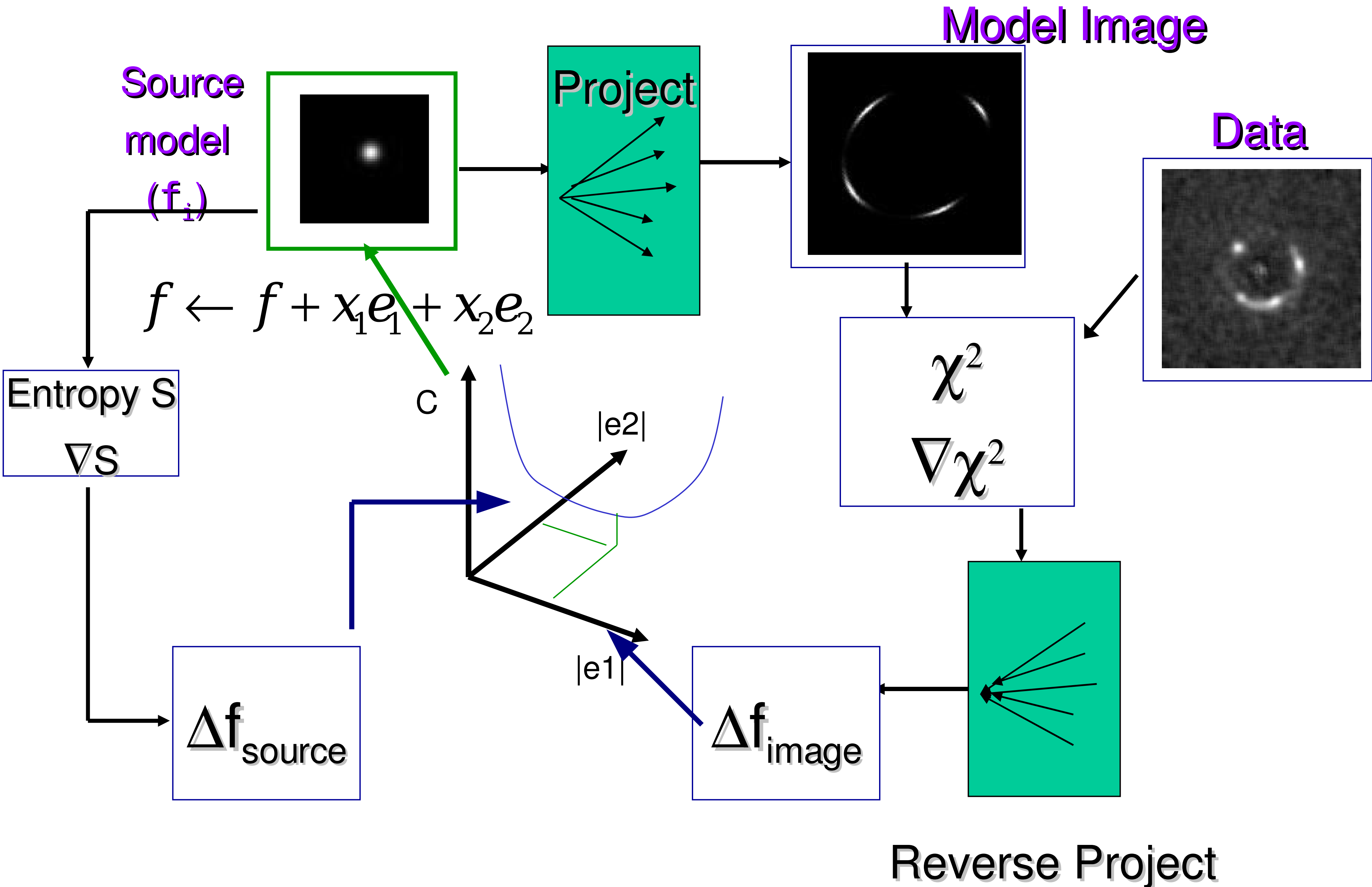
Wayth & Webster, 2006

- Based on LensMEM (Wallington et al 1994,1996)
- Non-parametric source (✔)
- Arbitrary PSF (✔)
- Arbitrary lens model (✔)
- Correctly preserves surface brightness (✔)

Iterative deconvolution - algorithm shematic



Lensview - algorithm shematic



Lensview- code for resolved lensed images

- ~9000 lines of C
- Uses Gnu Scientific Library, Cfitsio, FFTW
- Tested on Linux, Solaris, OS X
- Fast: deconvolution in ~1 sec
- Available from me (http://www.cfa.harvard.edu/~rwayth)

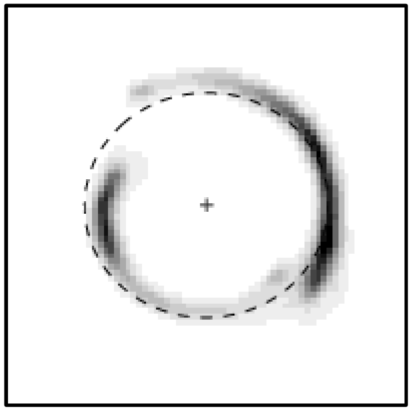
Statistics

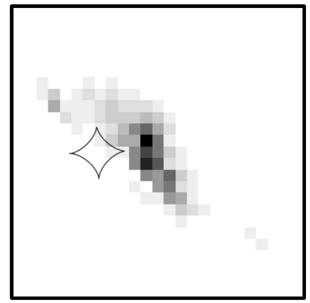
- N_{img} = number of image pixels
- N_{src} = number of source pixels
- m = number of params in mass model
- $\nu = N_{img} - N_{src} - m$ = degrees of freedom
- Target $\chi^2 = \nu$ +/- sqrt(2ν)
- χ^2 discriminates between mass models only if all other things are equal...

Statistics - practical considerations

- want to use as few img pixels as possible to maximize the discriminating power of χ^2
- use as few src pixels as possible while still retaining sufficient size and resolution
- should only count image pixels that map to the source plane
- But... the projected size of a fixed source plane changes in the image plane with mass model. Therefore an *image plane mask is essential*.

Example – changing projection of source plane





example source (enlarged scale)

mass model changed (larger Einstein radius)

source plane size enlarged

Statistics - practical considerations

- singly imaged pixels provide no constraints and should not be included in the mask
- pixels near center of image probably not very valuable due to demag and increased noise
- mass models can have substantially different magnification properties, the source plane should be adjusted accordingly

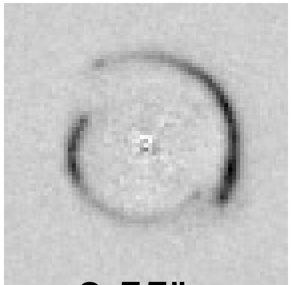
The mass model

- Potential components:
  - isothermal ellipsoids
  - point sources
  - external shear
  - power-law ellipsoids
  - Sersic constant M/L
  - NFW
  - user-defined constant M/L

Examples

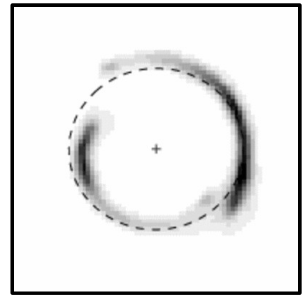
Example data courtesy of SLACS and Adam Bolton

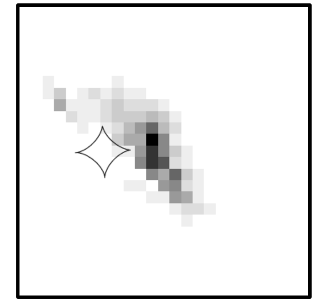
SDSSJ0252+0039



3.55”

neat image

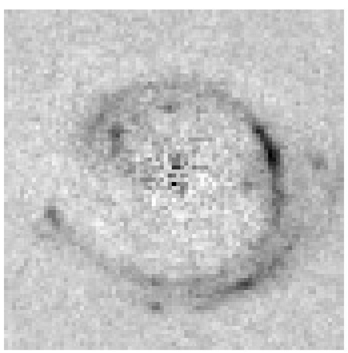




0.625”

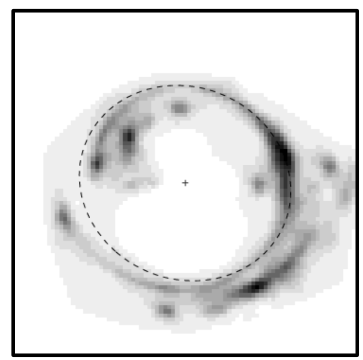
neat source

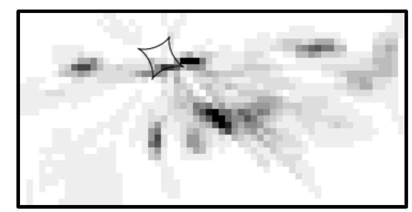
SDSSJ0728+3835



4.25”

messy image





1.5”

messy source

SLACS references:

Bolton et al. 2006

Treu et al. 2006

Koopmans et al. 2006

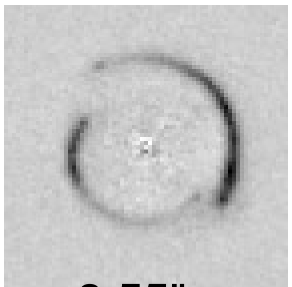
Gavazzi et al. 2007

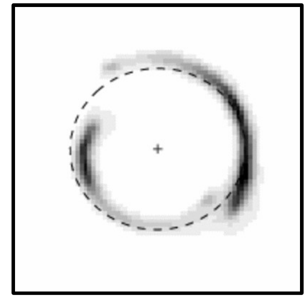
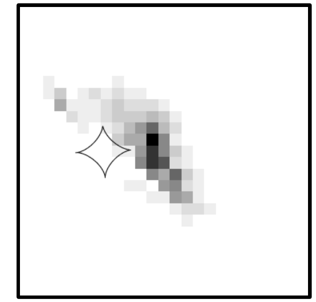
Bolton et al. in prep

Examples

The “Skipping Rope”

Example data courtesy of SLACS and Adam Bolton



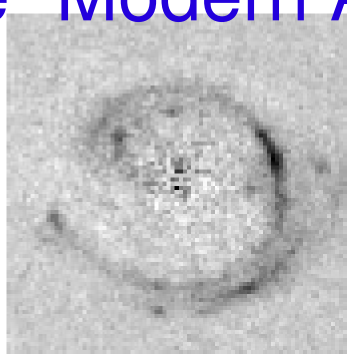
3.55”

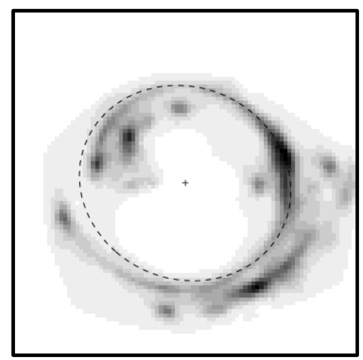
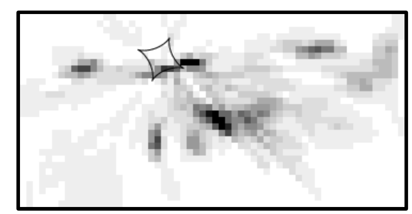
neat image

0.625”

neat source

The “Modern Art”



1.5”

messy source

4.25”

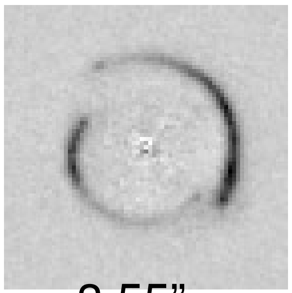
messy image

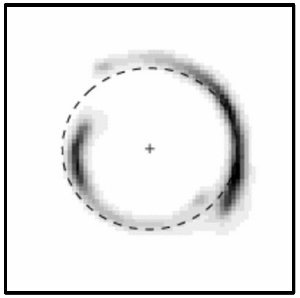
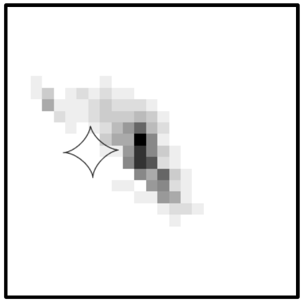
SLACS references:

Bolton et al. 2006
Treu et al. 2006
Koopmans et al. 2006
Gavazzi et al. 2007
Bolton et al. in prep

Examples

SDSSJ0252+0039



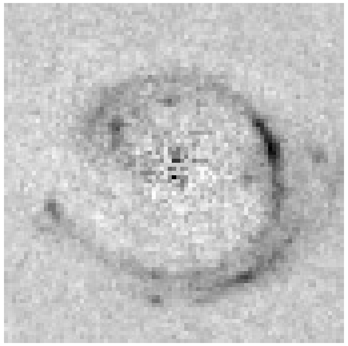
3.55”

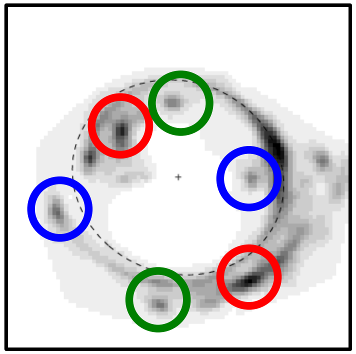
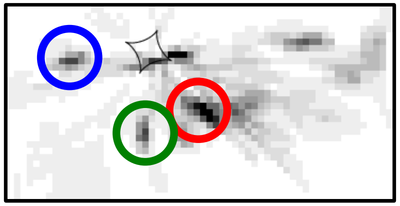
neat image

0.625”

neat source

SDSSJ0728+3835



1.5”

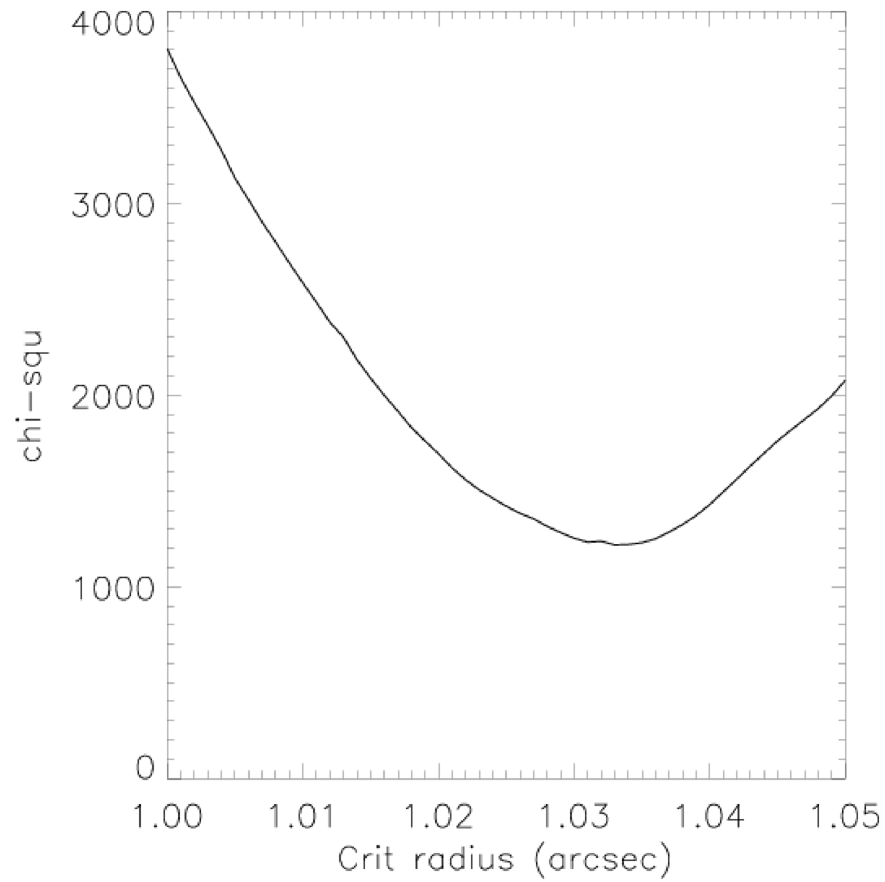
messy source

4.25”

messy image

Best fits and parameter confidence

- while technically not linear in params, χ^2 surface is well behaved
- parameter fit uncertainties can be estimated from marginalized χ^2 contours (Num Recipes chap 15)



Lensview tips

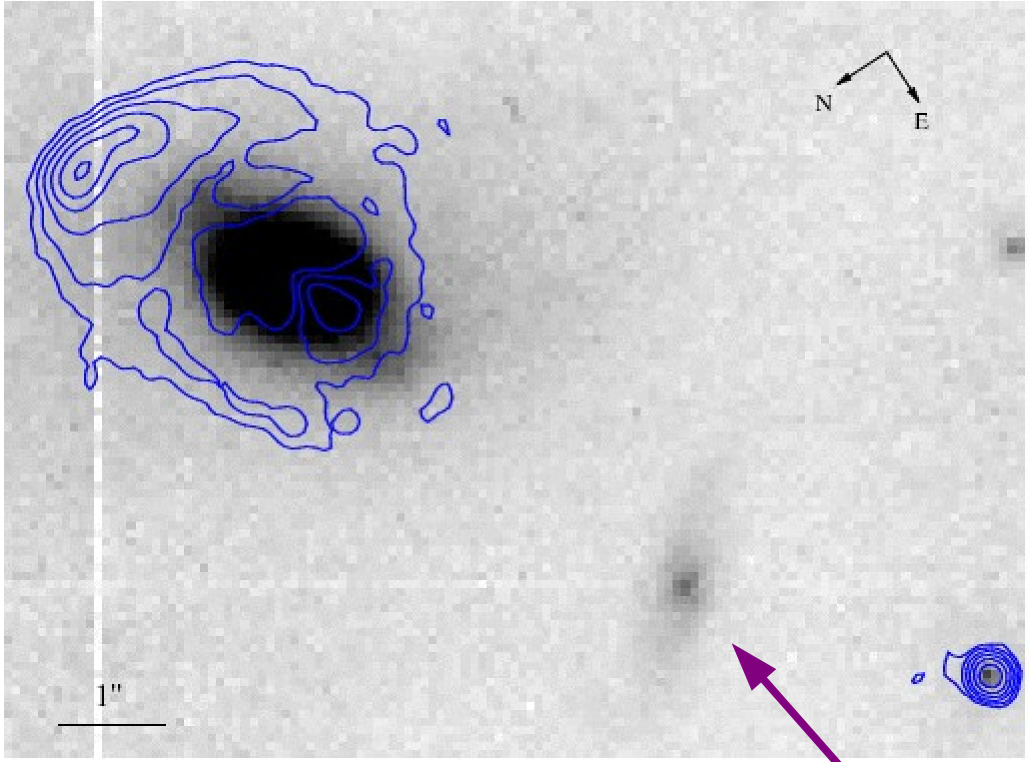
- Cut out small image from data, centered (within a pixel) on lens center.
  - minimizes pixel offset issues
  - minimizes amount of convolution for PSF
- Make initial guesstimate of lens properties
- Start with a (relatively) big source plane
- Use amoeba (minfinder) with modest search range
- Refine once initial model is secure
  - small source plane, higher mag etc.

Computational issues

- Parameter sweep is an “embarrassingly parallel” computational problem
- Lensview can calculate source and χ^2 for a set of params in ~1s
- CPU bound: Beowulf style cluster enables huge sweeps of parameter space
- Lens model components with analytic deflection angles are preferable

MG1549+3047

- z=1.17 radio lobe lensed by a z=0.11 SB0 galaxy



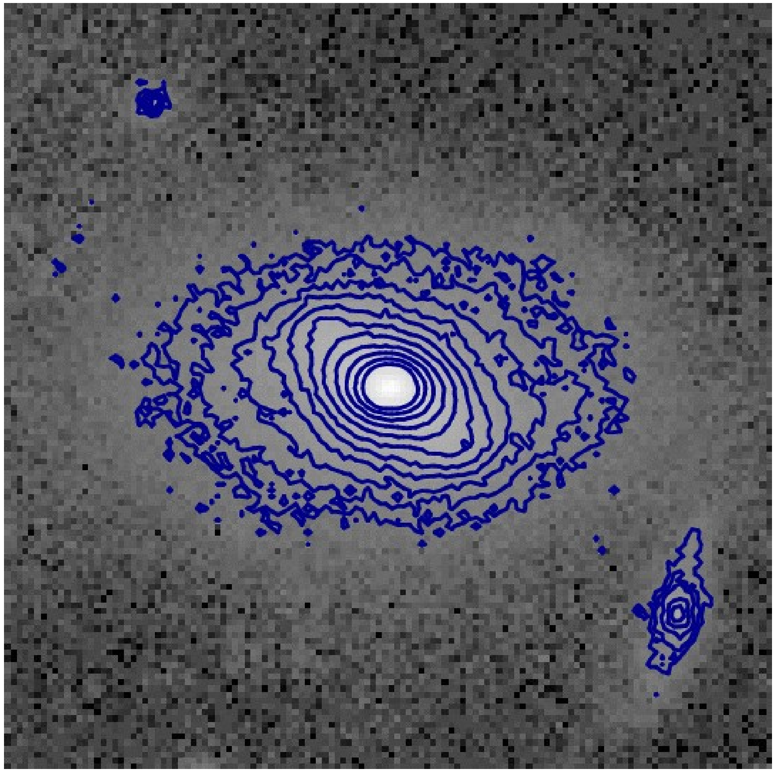
VLA 8.4GHz overlaid on NICMOS F160W.

Contours double from 0.2 mJy

z=0.6 intermediate galaxy

A constant M/L model for MG1549

- The complex structure of the lens galaxy is difficult to decompose.
- Need a way to calculate lensing deflection from the WFPC2 or NICMOS data



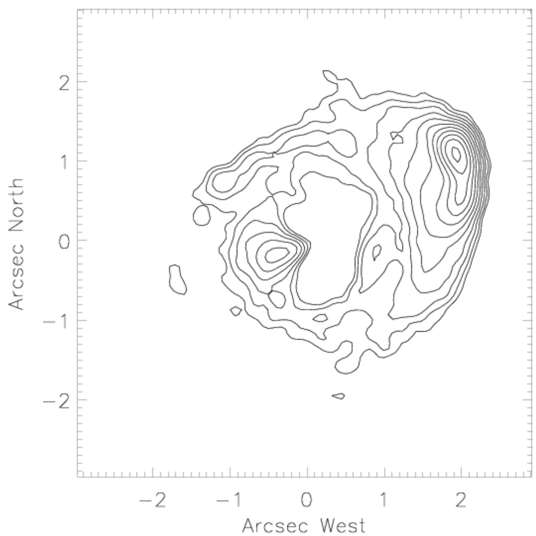
Recall lensing deflection equation:

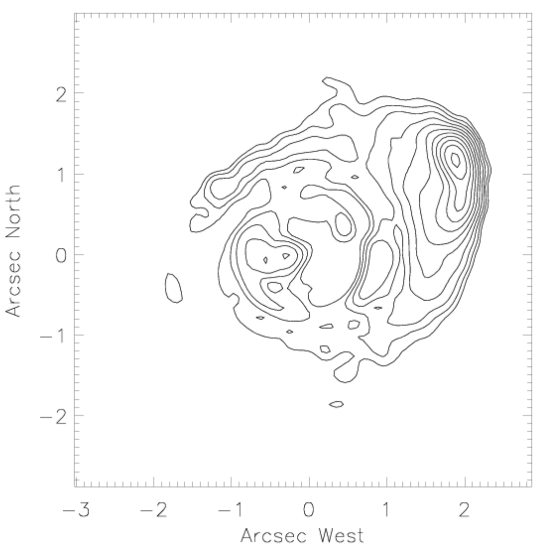
$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int \kappa(\vec{\theta}) \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2} d^2\theta'$$

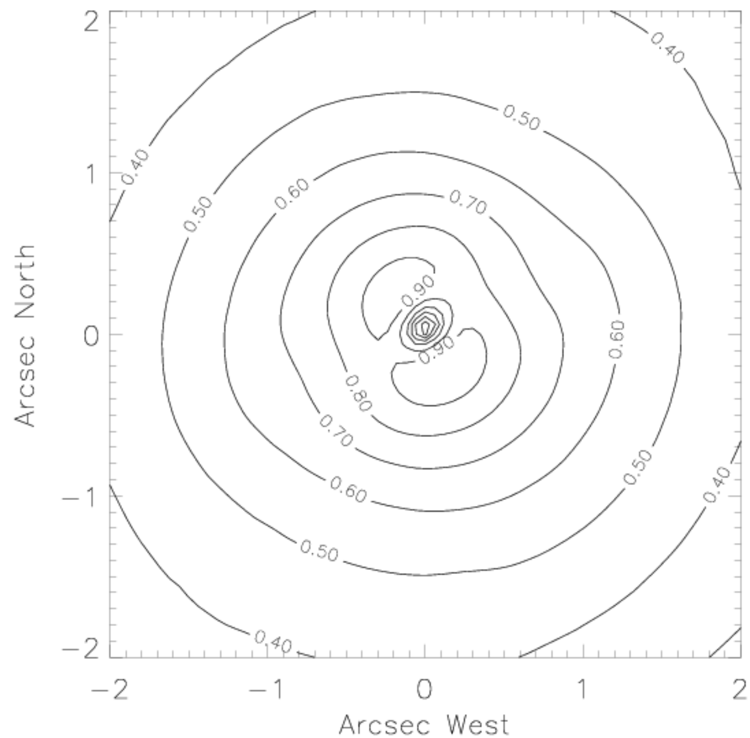
This is just a convolution of surface mass density $\kappa(\theta)$, with $1/\theta$.

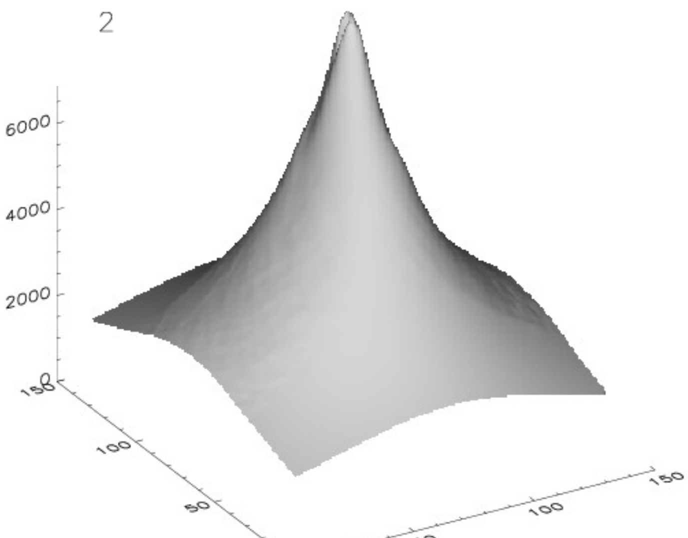
So... *we can use the NICMOS image to directly calculate deflections*.

A constant M/L model for MG1549





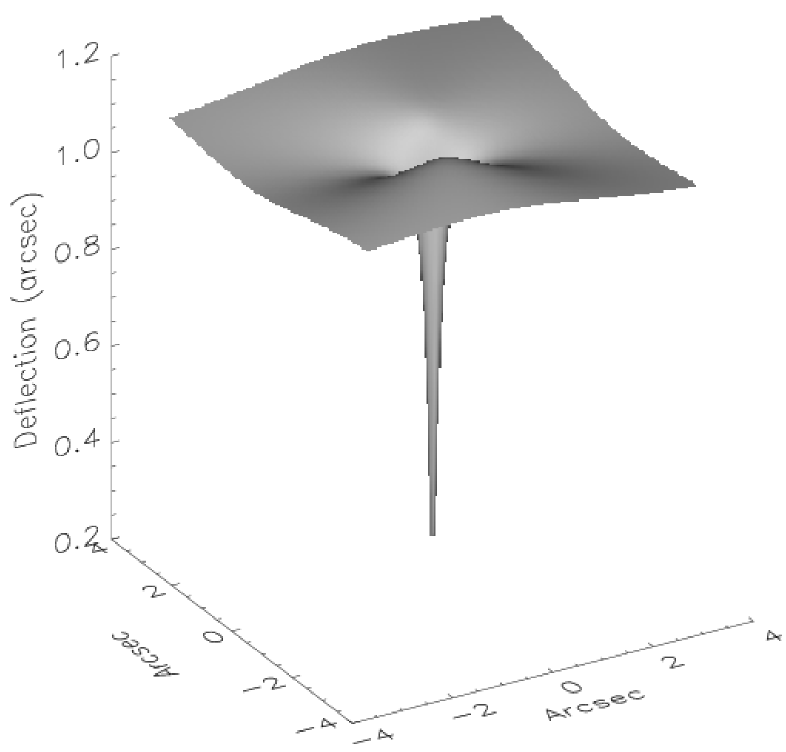




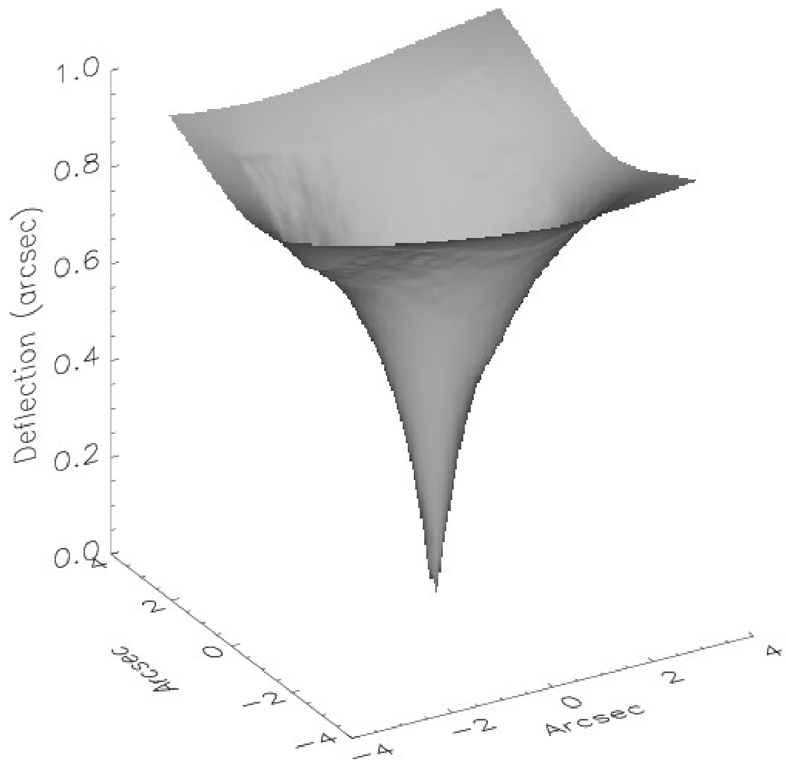
Magnitude of the deflection due to visible matter

The halo of MG1549

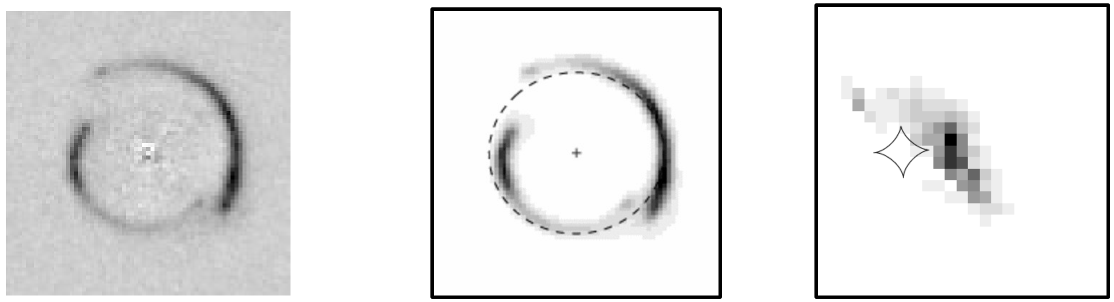
- Assuming the PIEP model is correct, we can directly subtract the stellar contribution to deflection



Magnitude of deflection for PIEP



Magnitude of deflection required of halo



Questions?