A CONVENIENT EXPRESSION FOR USE IN ANALYSIS OF CHERENKOV COUNTER DATA

DAVID CHRISTIAN

Fermi National Accelerator Laboratory
PO Box 500, Batavia, Il 60510

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A particle traveling faster than the speed of light in a medium with index of refraction n emits Cherenkov light with half angle given by

$$\cos\theta_c = \frac{1}{n\beta}. \tag{1}$$

In the approximation of zero dispersion (constant n), the number of photons emitted in a path length L (or with a different value of N_0, the number of photons detected) is

$$N = N_0 L \sin^2\theta_c. \tag{2}$$

This number is related to the number emitted (or detected) by a particle with $\beta = 1$ by

$$\frac{N}{N_{\max}} = \frac{\sin^2\theta_c}{\sin^2\theta_{\max}}. \tag{3}$$

Equation (3) can be rewritten in a form which is particularly convenient for use in the interpretation of Cherenkov counter data:

$$\frac{N}{N_{\max}} = \frac{\sin^2\theta_c}{\sin^2\theta_{\max}} = 1 - \frac{p_{th}^2}{p^2}, \tag{4}$$

where p_{th} = the lowest momentum (threshold) at which a specific type of particle emits Cherenkov light, and p = the particle's momentum.

This expression can be used directly to predict the amount of light emitted by a particle, or rearranged slightly to give the sine of the Cherenkov angle.

The derivation of (4) follows:

$$\sin^2\theta_c = 1-\cos^2\theta_c = \frac{n^2\beta^2-1}{n^2\beta^2} \tag{5}$$

$$\theta_c = \theta_{\max} \text{ when } \beta = 1;\ \sin^2\theta_{\max} = \frac{n^2-1}{n^2} \tag{6}$$

$$p^2 = (m\beta\gamma)^2 = \frac{m^2\beta^2}{1-\beta^2} \tag{7}$$

$$\text{At threshold, } \beta = \frac{1}{n} \Rightarrow p_{th}^2 = \frac{m^2/n^2}{1-\frac{1}{n^2}} = \frac{m^2}{\left(n^2-1\right)} \tag{8}$$

$$\begin{aligned}\frac{\sin^2\theta_c}{\sin^2\theta_{\max}} &= \frac{n^2\beta^2-1}{n^2\beta^2} \div \frac{n^2-1}{n^2} = \frac{n^2\beta^2-1}{n^2\beta^2-\beta^2} = \frac{\left(n^2\beta^2-\beta^2\right)+\left(\beta^2-1\right)}{n^2\beta^2-\beta^2}\\ &= 1-\frac{1-\beta^2}{\beta^2\left(n^2-1\right)} = 1-\left[\frac{1-\beta^2}{m^2\beta^2}\right]\left[\frac{m^2}{n^2-1}\right] = 1-\frac{p_{th}^2}{p^2}\end{aligned} \tag{9}$$

This can also be expressed in terms of γ:

$$p^2 = E^2 - m^2 \ \&\ E = m\gamma;\ p^2 = m^2\left(\gamma^2-1\right) \tag{10}$$

$$\begin{aligned}\frac{\sin^2\theta_c}{\sin^2\theta_{\max}} &= 1-\frac{p_{th}^2}{p^2} = \frac{p^2-p_{th}^2}{p^2} = \frac{m^2\left(\gamma^2-1\right)-m^2\left(\gamma_{th}^2-1\right)}{m^2\left(\gamma^2-1\right)}\\ &= \frac{\gamma^2-1-\gamma_{th}^2+1}{\gamma^2-1} = \frac{\gamma^2-\gamma_{th}^2}{\gamma^2-1}\end{aligned} \tag{11}$$

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