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Template track fitting

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Abstract

An algorithm is presented for non-iterative fitting of a track in magnetic field. Results of measurements of beam momenta using this fitting method are presented.

1 Introduction

Fitting trajectory of a particle through non-uniform magnetic field is computationally heavy, since in general one has to swim through magnetic field multiple times and in the case of iterative fitting, even on modern CPU's the fitting thousands of tracks takes a long time. If track measurements are made at given z positions, and track angles are not too large with respect to z-axis, non-iterative template track fitting works very well. In this case the required number of swims through magnetic field can be as small as one.

As charged particle traverses magnetic field, to first order its bend angle and therefore displacement from straight-line track is proportional to q/p, the charge and momentum of the particle. That is, at a given z, we can represent particle position with 5 parameters: $Q = q/p$, position (x, y) and direction $(dx/dz, dy/dz)$ at $z = z_0$:

$$\begin{cases} x = x_0 + \frac{dx}{dz}\Big|_{z=z_0} \cdot (z - z_0) + Q\lambda_x(z) \\ y = y_0 + \frac{dy}{dz}\Big|_{z=z_0} \cdot (z - z_0) + Q\lambda_y(z), \end{cases} \tag{1}$$

where λ_x and λ_y can be interpreted as displacement from straight-line trajectory for a particle with $q/p = 1$ and can be calculated from any track by swimming it from z_0 and setting λ to displacement times p/q. If track angles are not too large ($dx/dz < 1$) in the vicinity of z, then the errors resulting from this representation are negligible.

2 Fitting

Using track representation given by Equation 1, is especially appropriate for wire chambers, since the plane of the chamber is perpendicular to the z-axis. Coordinate u, perpendicular to wire direction in any given plane, is given by

$$u_i = x \cos\theta_i - y \sin\theta_i, \tag{2}$$

where angle θ_i between the y-axis and wires in the plane is measured clockwise with $\theta = 0$ for vertical wires. Thus, we can write prediction for u at z_i as

$$u(z_i) = \sum_j c_{ij} p_j, \tag{3}$$

where

$$\vec{p} = \begin{pmatrix} x_0 \\ y_0 \\ p_{x0} \\ p_{y0} \\ Q \end{pmatrix}, \qquad \vec{c_i} = \begin{pmatrix} \cos\theta_i \\ -\sin\theta_i \\ (z_i - z_0)\cos\theta_i \\ -(z_i - z_0)\sin\theta_i \\ \lambda_y(z_i)\cos\theta_i - \lambda_x(z_i)\sin\theta_i \end{pmatrix}.$$

To solve for $\vec{p}$, we write the sum of weighted residuals squared:

$$\chi^2 = \sum_i w_i (u_i - \sum_j c_{ij} p_j)^2, \tag{4}$$

and by defining

$$\begin{aligned} V_j &= \sum_i w_i u_i c_{ij} \\ M_{jk} &= \sum_i w_i c_{ij} c_{ik} \end{aligned}$$

solution to $\vec{p}$ which minimizes χ^2 is

$$\vec{p} = M^{-1}\vec{V}. \tag{5}$$

It follows that

$$\begin{aligned} \chi^2 &= \sum_i w_i u_i^2 - \vec{p}\cdot\vec{V} \\ \sigma_i^2 &= (M^{-1})_{ii} \end{aligned}$$

3 Application to beam tracks

Template track fit is perfectly suited to non-interacting beam tracks because a track has up to 34 measurement points from 9 chambers spanning more than 62 m and bend angles are quite small. In the Jolly Green Giant and Rosie

$$\int |B_y|\, dz \approx 1\text{Tm},$$

along the center of each magnet, so the lowest momentum 5 GeV/c beam tracks will get a transverse kick of ~ 65 mrad.

My approach to the fit was to use seed λ's calculated with

$$\begin{pmatrix} x \\ y \\ dx/dz \\ dy/dz \\ q/p \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1/120 \end{pmatrix}$$

to estimate track parameters, then recalculate λ's by swimming the track through the experiment using the current guess at parameters and refit. Typically, λ's used for the final fit differ from the seed values by less than 1% regardless of track direction and momentum.

4 Results

One of the first tasks completed with fast fit was tuning the relative magnet strengths in software. Although field in both magnets is measured with Hall probes (one above the bottom coil and one below the top coil), their locations were not known precisely. While Rosie field is sufficiently uniform in the vicinity of Hall probes, JGG field is very non-uniform in the vicinity of the Hall probes, and as Figure 1 shows, all momentum measurements came $5 - 10\%$ high.

Signs that something was wrong included a systematically smaller momentum measurement if the first two beam chambers (with the largest lever arm) were excluded from the shift. Ideally, one would not expect the central value of momentum measurement to shift if any one detector was taken out of the measurement. Second sign was clear dependence of momentum on χ^2 per degree of freedom of the track.

To find out whether magnet strengths are not correctly matched in the software, a scan of JGG scaling factor was done. The most obvious metrics are number of tracks with χ^2 per degree of freedom less than 0.4 and the average χ^2 of all tracks, shown in Figure 3.

Once chamber wire planes were aligned, we measured momentum for each run and agreement between the measured momentum and beamline setting is very good, as shown on Figure 4.

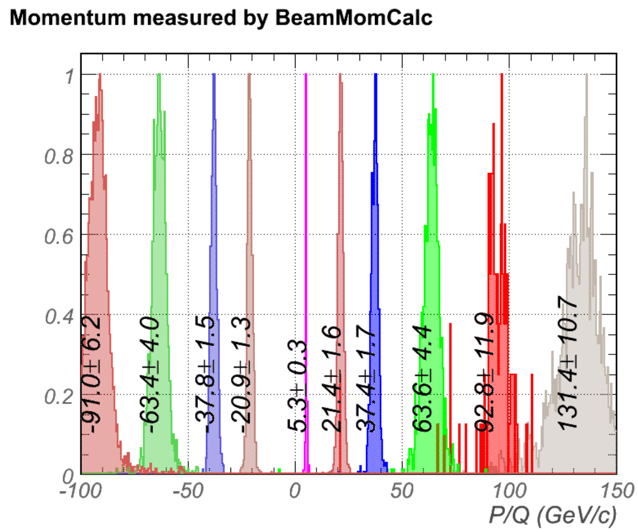
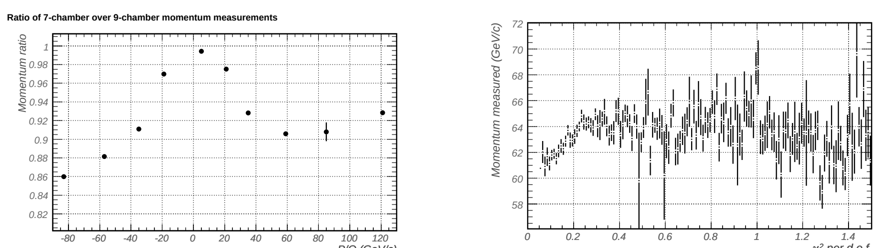


Figure 1: Initial measurement of beam momenta. Besides the 120 GeV/c primary, $5, \pm 20, \pm 35, \pm 58$, and ± 84 GeV/c secondary beams were measured.



a) *b)*

Figure 2: *a)* Ratio of momentum measurements with 7 and 9 chambers. *b)* Dependence of momentum measurement on χ^2 per degree of freedom for +58 GeV/c run.

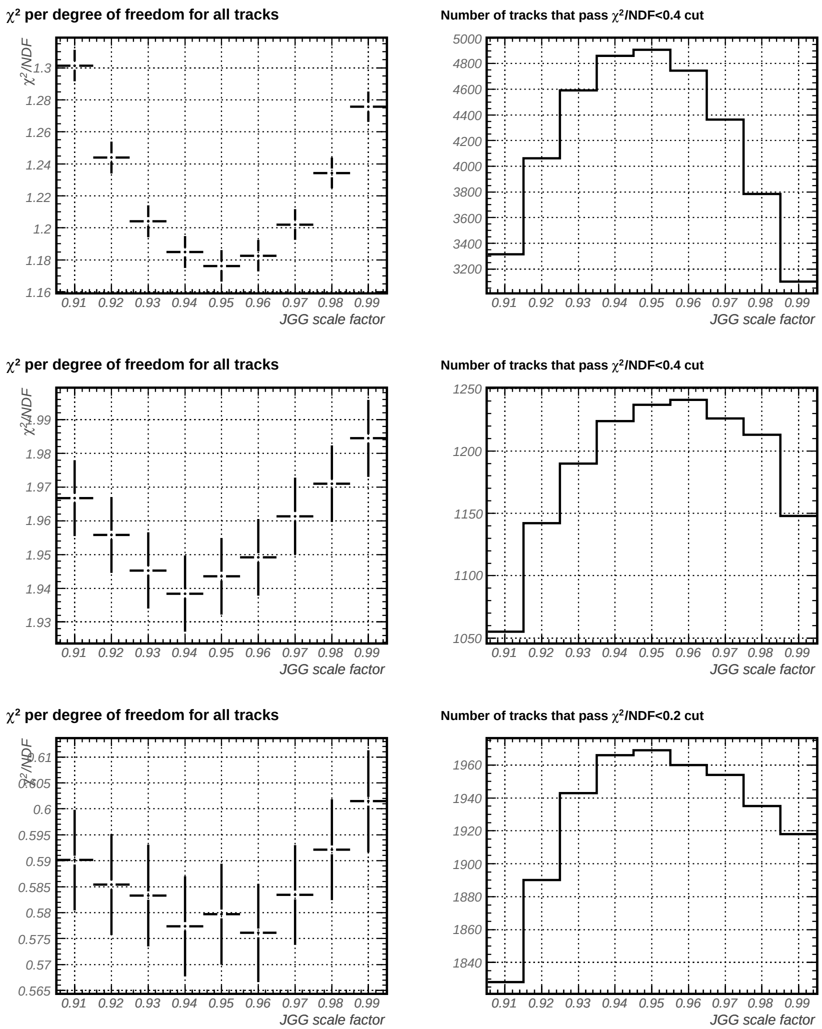


Figure 3: JGG scale factor scan results. From top to bottom -35 GeV/c, $+58$ GeV/c, $+120$ GeV/c runs.

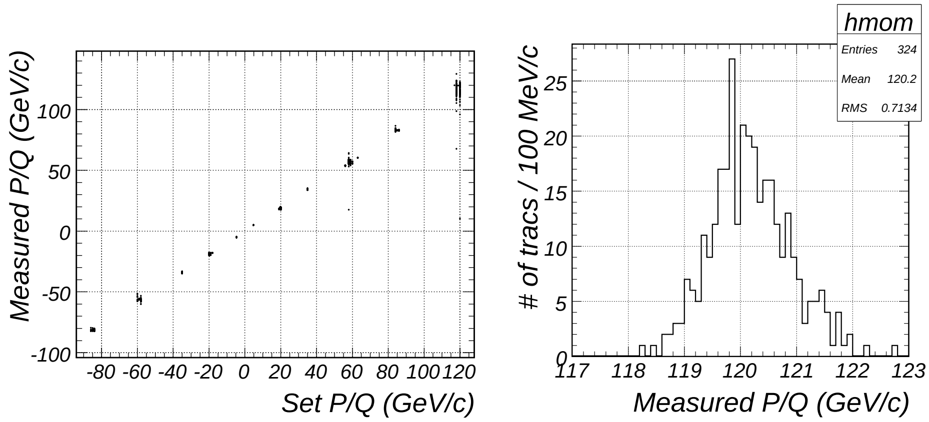


Figure 4: Measured momentum vs beamline setting for 2600 runs (left) and 120 GeV/c momentum measurement (right).

5 Conclusion

Template track fit is a fast, non-iterative track fitting method that is especially applicable to fitting tracks with small angles with respect to z-axis. If a good set of seed coefficients can be provided, then only one swim per track is sufficient to obtain a reliable track fit.

While this note describes using this method for fitting tracks with data from wire chambers, there is no reason why the method will not work with other tracking detectors, in particular the time projection chamber. The only difference will be that every 3-dimensional TPC measurement will have to be represented as two 2-dimensional measurements (x_i at z_i and y_i at z_i) with corresponding $\theta_x = 0$ and $\theta_y = 90°$. Thus the fitting algorithm can be used for global tracking for those tracks where dz/ds does not change sign.